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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Intermediate macroeconomics

Course Code: ECC2010 • Module 3 Notes

Module 3: Microfoundations: Intertemporal Consumption, Investment & Labour Markets

Chapter 1: The Fisher Two-Period Intertemporal Consumption Model

1.1 Microfoundations and the Intertemporal Budget Constraint

Traditional Keynesian consumption functions specify current consumption solely as a mechanical function of current disposable income ($C = c_0 + c_1 Y_d$). Following the Lucas critique (1976), modern macroeconomics derives aggregate behavioural equations from explicit forward-looking optimisation by households and firms. Irving Fisher's (1930) two-period model forms the foundation of intertemporal consumption theory.

Consider a representative household living for two periods: Period 1 (the present/working period) and Period 2 (the future/retirement period). The household begins Period 1 with initial financial wealth A_0, receives exogenous real disposable incomes Y_1 and Y_2 in Periods 1 and 2, chooses consumptions C_1 and C_2, and can borrow or lend freely at a uniform real interest rate r. Period 1 saving is $S_1 = A_0 + Y_1 - C_1$, and Period 2 consumption equals Period 2 income plus gross principal and interest on Period 1 saving:

$$C_2 = Y_2 + (1 + r)(A_0 + Y_1 - C_1)$$

Dividing both sides by $(1 + r)$ and rearranging places all consumption on the left and lifetime resources on the right, giving the **intertemporal budget constraint**:

$$C_1 + \frac{C_2}{1 + r} = A_0 + Y_1 + \frac{Y_2}{1 + r} = W_1$$

Where: C_1 and C_2 are real consumption in Periods 1 and 2; r is the real interest rate; A_0 is initial non-human wealth; Y_1 and Y_2 are disposable labour incomes; and W_1 is total lifetime wealth (present discounted value of lifetime resources). The slope of the budget line in (C_1, C_2) space is $-(1 + r)$, representing the opportunity cost of consuming one additional unit today in terms of foregone future consumption.

1.2 Preferences, the Euler Equation, and Consumption Smoothing

The household maximises lifetime expected utility $U(C_1, C_2) = u(C_1) + \beta u(C_2)$ subject to the intertemporal budget constraint, where $\beta = \frac{1}{1+\rho} \in (0, 1)$ is the subjective discount factor ($\rho > 0$ is the pure rate of time preference) and $u(\cdot)$ is strictly increasing and strictly concave ($u'(C) > 0, u''(C) < 0$). Concavity implies diminishing marginal utility and creates a fundamental motive for **consumption smoothing** across periods.

Setting the marginal rate of substitution (MRS_{C_1,C_2}) equal to the relative price $(1 + r)$ yields the discrete-time **consumption Euler equation**:

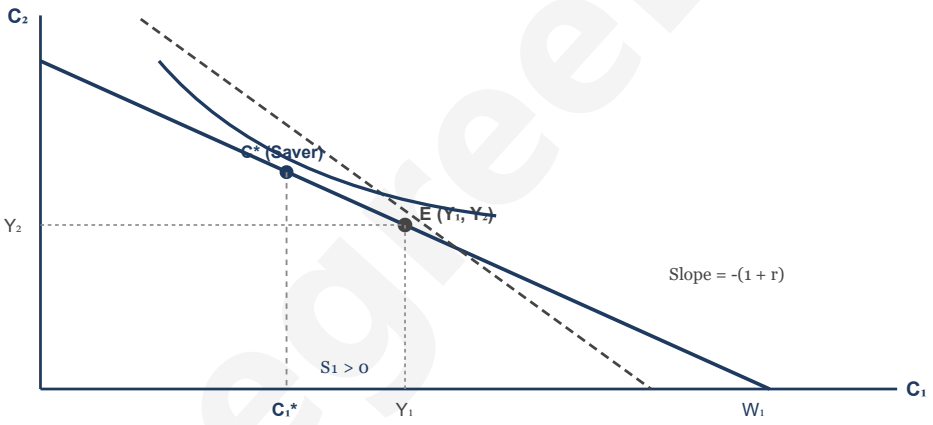
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$$\frac{u'(C_1)}{\beta u'(C_2)} = 1 + r \quad \Leftrightarrow \quad u'(C_1) = \beta(1 + r)u'(C_2)$$

Where: $u'(C_t)$ is the marginal utility of consumption in period t; $\beta = 1/(1 + \rho)$ is the patience discount factor; and $1 + r$ is the gross real return on saving. Under Constant Relative Risk Aversion (CRRA) utility $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$ (which reduces to logarithmic utility $u(C) = \ln C$ as $\sigma \to 1$), where $\sigma > 0$ is the coefficient of relative risk aversion and $1/\sigma$ is the **elasticity of intertemporal substitution (EIS)**, the Euler equation simplifies to:

$$\frac{C_2}{C_1} = [\beta(1 + r)]^{1/\sigma} = \left(\frac{1 + r}{1 + \rho}\right)^{1/\sigma}$$

Where: $1/\sigma$ governs how strongly households tilt consumption toward the future when r rises above ρ. If $r = \rho$ (so $\beta(1 + r) = 1$), optimal consumption is perfectly flat ($C_1 = C_2$); if $r > \rho$, consumption grows over time ($C_2 > C_1$); and if $r < \rho$, consumption declines over time ($C_2 < C_1$).



1.3 Income, Substitution, and Wealth Effects of Real Interest Rate Changes

When the real interest rate r rises (for instance, due to RBA monetary tightening), the intertemporal budget line pivots clockwise through the endowment point $(Y_1 + A_0, Y_2)$. Decomposing the response of Period 1 consumption (C_1) requires separating two opposing forces:

- **Intertemporal Substitution Effect:** A higher r makes current consumption C_1 relatively more expensive compared to future consumption C_2 (each dollar saved now yields $1 + r$ tomorrow). This substitution effect unambiguously *reduces* C_1 and raises saving for both savers and borrowers.
- **Income (Endowment) Effect:** For a *net saver* ($C_1 < A_0 + Y_1$), higher r raises interest income on existing savings, expanding feasible consumption in both periods and pushing C_1 *upward*. For a *net borrower* ($C_1 > A_0 + Y_1$), higher r increases debt-service costs, lowering lifetime real resources and reinforcing the substitution effect to push C_1 sharply *downward*.

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Consequently, a rise in r unambiguously lowers current consumption for borrowers, whereas for savers the net impact depends on the EIS ($1/\sigma$): under log utility ($\sigma = 1$), the income and substitution effects on C_1 cancel exactly when $Y_2 = 0$, leaving C_1 unchanged; when $\sigma < 1$, substitution dominates and saving rises; when $\sigma > 1$ (empirical estimates often place $\sigma \approx 2$), the income effect dominates for substantial asset holders.

1.4 Borrowing Constraints and Interest Rate Wedges

In real financial markets, households cannot always borrow freely against future labour income (Y_2). If a household faces a binding **liquidity (borrowing) constraint** $C_1 \leq A_0 + Y_1$, or a higher borrowing rate $r_b > r_l$, young households expecting high future income cannot smooth consumption upward and instead consume their entire current cash-on-hand ($C_1 = A_0 + Y_1$). Liquidity-constrained households exhibit a high marginal propensity to consume (MPC) out of current income and temporary transfers—a crucial channel for fiscal stimulus effectiveness.

Worked Example 1: Optimal Intertemporal Choice, Interest Rate Hike, and Credit Constraints

Consider an Australian graduate with initial wealth $A_0 = 0$, current income $Y_1 = 60{,}000$ AUD, expected future income $Y_2 = 96{,}000$ AUD, and logarithmic utility $U = \ln C_1 + \beta \ln C_2$ with discount factor $\beta = 0.90$.

1. **Unconstrained optimum at $r = 0.05$ (5%):** Lifetime wealth is:

$$W_1 = 60{,}000 + \frac{96{,}000}{1.05} = 60{,}000 + 91{,}428.57 = 151{,}428.57 \text{ AUD}$$

With log utility ($\sigma = 1$), the Euler equation gives $C_2 = \beta(1 + r)C_1$. Substituting into $C_1 + \frac{C_2}{1+r} = W_1$ yields $C_1 + \beta C_1 = W_1 \implies C_1^* = \frac{W_1}{1+\beta}$:

$$C_1^* = \frac{151{,}428.57}{1 + 0.90} = 79{,}699.25 \text{ AUD}$$

$$C_2^* = 0.90 \times 1.05 \times 79{,}699.25 = 75{,}315.79 \text{ AUD}$$

Because $C_1^* = 79{,}699.25 > Y_1 = 60{,}000$, the household borrows $19{,}699.25$ AUD in Period 1 ($S_1 = -19{,}699.25$).

2. **Impact of an interest rate hike to $r' = 0.20$ (20%):** Lifetime wealth falls to $W_1' = 60{,}000 + 96{,}000/1.20 = 140{,}000$ AUD. Optimal Period 1 consumption falls to:

$$C_1^{*\prime} = \frac{140{,}000}{1.90} = 73{,}684.21 \text{ AUD}, \qquad C_2^{*\prime} = 0.90 \times 1.20 \times 73{,}684.21 = 79{,}578.95 \text{ AUD}$$

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Both the substitution effect and the negative income effect (as a borrower) lower C_1 by $6{,}015.04$ AUD.

3. **Binding borrowing constraint ($C_1 \leq Y_1$):** If banks refuse unsecured loans ($S_1 \geq 0$), the unconstrained optimum $79{,}699.25$ is infeasible, so the household is cornered at $C_1 = 60{,}000$ AUD and $C_2 = 96{,}000$ AUD. Any temporary Period 1 bonus of $5{,}000$ AUD raises C_1 one-for-one ($MPC = 1.0$) rather than by $\frac{1}{1.90} = 0.526$.

Chapter 2: Permanent Income, Life-Cycle & Random Walk Consumption

2.1 Friedman's Permanent Income Hypothesis (PIH)

Extending Fisher's model to an infinite or multi-period horizon T, Milton Friedman (1957) decomposed observed income (Y_t) into a persistent **permanent income** component (Y_t^P, the constant annuity value of lifetime wealth) and a short-lived **transitory income** component (Y_t^T):

$$Y_t = Y_t^P + Y_t^T, \qquad C_t = Y_t^P = \frac{r}{1+r}\left[A_t + \sum_{s=0}^{\infty} \frac{E_t[Y_{t+s}]}{(1+r)^s}\right]$$

Where: Y_t^P is permanent income (assuming $\beta(1+r) = 1$); Y_t^T is transitory income; A_t is current financial wealth; and $E_t[Y_{t+s}]$ is expected future labour income. The central prediction of the PIH is the sharp divergence in marginal propensities to consume:

- **Permanent Income Shock (ΔY_t^P):** An unanticipated, permanent salary promotion raises Y_t^P one-for-one, so $MPC_{\text{perm}} \approx 1.0$: consumption shifts upward permanently.
- **Transitory Income Shock (ΔY_t^T):** A one-off tax bonus or temporary overtime windfall raises lifetime wealth only by ΔY_t^T, increasing the sustainable annuity Y_t^P by merely $\frac{r}{1+r}\Delta Y_t^T$ (or $1/T$ over a T-year horizon). Thus $MPC_{\text{temp}} \ll 1$, and almost all transitory income is saved.

2.2 Hall's Random Walk Hypothesis and Empirical Puzzles

Robert Hall (1978) combined the Euler equation with **rational expectations**. Under quadratic utility (or CRRA utility with $\beta(1+r) = 1$ and small shocks), the stochastic Euler equation $u'(C_t) = \beta(1+r)E_t[u'(C_{t+1})]$ implies:

$$E_t[C_{t+1}] = C_t \quad \Leftrightarrow \quad C_{t+1} = C_t + \varepsilon_{t+1}, \qquad E_t[\varepsilon_{t+1}] = 0$$

Where: ε_{t+1} is an unpredictable consumption innovation driven purely by *new information* about lifetime income arriving between t and $t+1$. Predictable changes in income (such as scheduled award wage increments or anticipated tax cuts announced months earlier) should already be incorporated into C_t and therefore produce zero predictable change in C_{t+1}.

Empirical microdata and aggregate ABS tests frequently uncover three departures from the pure certainty-equivalent PIH: (1) **excess sensitivity** of consumption to predictable income changes (caused by liquidity constraints and rule-of-thumb consumers); (2) **excess smoothness** of aggregate

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consumption relative to measured permanent income shocks; and (3) **precautionary saving**, whereby prudent households ($u'''(C) > 0$) facing uninsurable labour income risk accumulate buffer-stock wealth beyond certainty-equivalent predictions.

2.3 Modigliani's Life-Cycle Hypothesis and Australian Retirement Saving

Franco Modigliani (1954) emphasised deterministic age-profile earnings over a finite horizon of T remaining years, comprising R working years (earning Y) and $T - R$ retirement years (earning zero labour income). With $r = \rho = 0$ and zero initial assets, smoothing consumption evenly across T years gives constant annual consumption:

$$C = \frac{R \cdot Y + A_0}{T}$$

Where: R is remaining working years; T is total remaining lifespan; Y is annual labour income; and A_0 is initial net wealth. Households save $S = Y - C = \frac{T-R}{T} Y$ during working life—building a hump-shaped wealth profile peaking at retirement—and dissave ($-C$) during retirement. In Australia, this life-cycle mechanism is institutionalised through the three-pillar retirement system: the means-tested Commonwealth Age Pension, mandatory employer contributions under the Superannuation Guarantee (12% by July 2025), and voluntary private superannuation and housing equity.

Worked Example 2: Life-Cycle Consumption and Wealth Accumulation

Suppose an Australian worker begins full-time employment at age 25 with initial assets $A_0 = 0$, expects to work for $R = 40$ years (retiring at 65) with constant real net income $Y = 85{,}000$ AUD per year, and expects to live until age 85 ($T = 60$ total adult years). Assume $r = \rho = 0$ for tractability.

1. **Constant life-cycle consumption (C):** Total lifetime earnings equal $40 \times 85{,}000 = 3{,}400{,}000$ AUD. Smoothing across $T = 60$ years gives:

$$C = \frac{3{,}400{,}000}{60} = 56{,}666.67 \text{ AUD per year}$$

2. **Annual saving and peak retirement wealth:** During each working year, the worker saves $S = 85{,}000 - 56{,}666.67 = 28{,}333.33$ AUD (33.3% of net income). By retirement at age 65, accumulated wealth peaks at $40 \times 28{,}333.33 = 1{,}133{,}333.33$ AUD—exactly financing $20 \times 56{,}666.67 = 1{,}133{,}333.33$ AUD of retirement consumption.
3. **Impact of an unexpected $10{,}000$ AUD one-off bonus at age 25:** The marginal propensity to consume out of this transitory windfall is $MPC = 1/60 = 0.0167$, raising annual consumption by only 166.67 AUD per year and saving $9{,}833.33$ AUD in Year 1.

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Chapter 3: Fiscal Policy Microfoundations & Ricardian Equivalence

3.1 The Intertemporal Government Budget Constraint and Barro-Ricardo Theorem

In the two-period Fisher framework, consider a government that begins with initial real public debt B_0, purchases G_1 and G_2 in Periods 1 and 2, and levies lump-sum taxes T_1 and T_2. Period 1 budget deficit $B_1 - B_0 = G_1 + rB_0 - T_1$ must be repaid in Period 2 via $T_2 = G_2 + (1+r)B_1$. Eliminating B_1 yields the **intertemporal government budget constraint**:

$$G_1 + \frac{G_2}{1+r} + (1+r)B_0 = T_1 + \frac{T_2}{1+r}$$

Where: G_1, G_2 are real government purchases; T_1, T_2 are lump-sum taxes; B_0 is initial public debt; and r is the real interest rate on government bonds. Now substitute the present value of taxes $T_1 + \frac{T_2}{1+r}$ from the government constraint into the representative household's lifetime budget constraint (where Y_t is pre-tax income minus T_t):

$$C_1 + \frac{C_2}{1+r} = A_0 + (Y_1 - G_1) + \frac{Y_2 - G_2}{1+r} - (1+r)B_0$$

This identity establishes the **Ricardian Equivalence Theorem** (David Ricardo 1820; Robert Barro 1974): holding the path of government purchases (G_1, G_2) fixed, a debt-financed lump-sum tax cut ($\Delta T_1 < 0$ financed by issuing bonds $\Delta B_1 = -\Delta T_1$) leaves household lifetime wealth *unchanged*, because forward-looking taxpayers recognise that the tax cut today must be offset by higher future taxes $\Delta T_2 = -(1+r)\Delta T_1$ tomorrow. Consequently, households save 100% of the tax cut to buy the newly issued government bonds ($\Delta S_p = -\Delta S_g$), leaving aggregate consumption (C_1), national saving ($S = S_p + S_g$), real interest rates (r), and output (Y) completely unchanged.

3.2 Five Conditions Causing Ricardian Equivalence to Fail

In practice, empirical studies typically find that 30% to 60% of a temporary tax cut is saved rather than 100%. Ricardian equivalence breaks down whenever any of its strict assumptions are violated:

1. **Binding Borrowing Constraints:** Credit-constrained households discount future taxes at a higher effective shadow rate than the government's borrowing rate r. A deficit-financed tax cut acts like a low-interest loan from the government, immediately boosting C_1.
2. **Finite Horizons Without Operative Altruistic Bequests:** If current taxpayers die before future taxes T_2 are collected and do not value their descendants' utility equally via Barro-style dynastic bequests, shifting taxes to future generations raises current cohorts' lifetime wealth.
3. **Distortionary Taxation:** Real-world income, corporate, and GST taxes distort labour supply and investment incentives, altering Y_1 and Y_2 when tax rates change over time.
4. **Myopia and Bounded Rationality:** Households may overlook future tax liabilities or treat government bonds as net private wealth.

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5. **Uncertain Future Income:** When future taxes are proportional to uncertain future income, a lump-sum tax cut combined with proportional future taxes reduces individual income variance, lowering precautionary saving and raising C_1.

Worked Example 3: Testing Ricardian Equivalence with Heterogeneous Households

Suppose an economy comprises 100 identical households, each earning pre-tax incomes $Y_1 = Y_2 = 100$ (in thousand AUD), $A_0 = 0$, $B_0 = 0$, $r = 0.10$, and log utility with $\beta = 1/1.10$ (so $C_1 = C_2$ when unconstrained). Government purchases are $G_1 = G_2 = 20$ per household, initially funded by balanced budgets $T_1 = T_2 = 20$. Thus initial $C_1 = C_2 = 80$, private saving $S_p = 0$, and public saving $S_g = 0$.

1. **Pure Ricardian benchmark (all households unconstrained):** The government cuts Period 1 taxes to $T_1' = 10$ ($\Delta T_1 = -10$ per household), issuing 10 in bonds per household. Period 2 taxes must rise to $T_2' = 20 + 1.10(10) = 31$. New after-tax incomes are $(90, 69)$. Lifetime wealth is $W_1' = 90 + 69/1.10 = 90 + 62.73 = 152.73$, identical to baseline $80 + 80/1.10 = 152.73$. Optimal $C_1' = 80$: private saving jumps from 0 to $+10$, exactly offsetting the public deficit of -10, so national saving remains 0.
2. **Mixed economy (40% credit-constrained, 60% Ricardian):** The 60 Ricardian households save their entire 10 tax cut ($\Delta C_1 = 0$), while the 40 borrowing-constrained households have $MPC = 1.0$ out of current cash flow and spend their entire 10 tax cut ($\Delta C_1 = +10$ each). Aggregate consumption rises by $40 \times 10 = 400$ thousand AUD (or 4.0 per household on average), aggregate private saving rises by only 600 (failing to offset the $1{,}000$ government deficit), and national saving falls by 400—raising real interest rates and crowding out investment.

Chapter 4: Neoclassical Investment Theory & Tobin's q

4.1 Jorgenson's User Cost of Capital and Desired Capital Stock

Because business fixed investment is the most volatile component of GDP, its microeconomic foundations are central to both long-run growth and business cycles. Dale Jorgenson (1963) modelled a firm choosing its optimal capital stock (K^*) by comparing the real marginal product of capital (MPK) against the **real user cost of capital** (u_K / P). Suppose a firm purchases one unit of capital at nominal price P_K, finances it at nominal interest rate i, suffers physical depreciation δ, and expects the capital good's price to appreciate by $\Delta P_K^e / P_K$. The real user cost per unit of capital is:

$$u_K = \left(i - \frac{\Delta P_K^e}{P_K} + \delta\right) \frac{P_K}{P} \approx (r + \delta)\, p_K$$

Where: u_K is the real user cost of capital; $r = i - \pi^e$ is the real interest rate; δ is physical depreciation; and $p_K = P_K / P$ is the relative price of capital goods (if capital goods prices rise at overall inflation π^e). Incorporating corporate income taxation at rate τ, the present value of depreciation tax shields z, and an investment tax credit k_{ITC} yields the **tax-adjusted user cost**:

$$u_K^{\text{tax}} = \frac{(r+\delta)(1-\tau z - k_{ITC})}{1-\tau} p_K$$

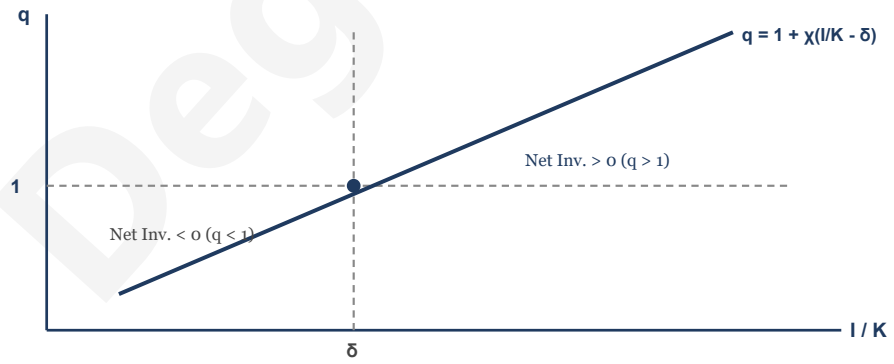
Where: τ is the statutory corporate tax rate (e.g., 30% in Australia for large firms); z is the present discounted value of depreciation allowances per dollar of investment (equal to 1 under instant asset write-off); and k_{ITC} is any investment tax credit. Profit maximisation sets $MPK(K^*) = u_K^{\text{tax}}$, yielding desired capital K^* under Cobb-Douglas $Y = AK^{\alpha}L^{1-\alpha}$ as $K^* = \frac{\alpha Y}{u_K^{\text{tax}}}$. Net investment responds gradually via a partial-adjustment or flexible-accelerator rule $I_t = \lambda(K^* - K_{t-1}) + \delta K_{t-1}$.

4.2 Convex Adjustment Costs and Tobin's q Theory

If capital could be adjusted instantaneously without cost, investment would jump infinitely whenever r changed. In reality, installing machinery, retraining workers, and constructing factories entail **convex adjustment costs** $\Phi(I_t, K_t) = \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$, where $\chi > 0$ measures the severity of installation frictions. Maximising the firm's stock-market value V_t subject to $K_{t+1} = (1-\delta)K_t + I_t$ equates the marginal cost of buying and installing one extra unit of capital to its shadow marginal value inside the firm (q_t, defined by James Tobin, 1969):

$$\$1 + \chi\left(\frac{I_t}{K_t} - \delta\right) = q_t \quad \Longrightarrow \quad \frac{I_t}{K_t} = \delta + \frac{q_t - 1}{\chi}\$$

Where: q_t is **marginal Tobin's** q (the ratio of the market value of an additional unit of installed capital to its replacement cost p_K), approximated empirically by **average** q ($q_t^{\text{avg}} = \frac{\text{Equity Market Cap} + \text{Debt}}{p_K K_t}$); I_t/K_t is the gross investment-to-capital ratio; δ is depreciation; and χ is the adjustment cost parameter. When $q_t > 1$, markets value installed capital above its replacement cost, inducing positive net investment ($I_t/K_t > \delta$); when $q_t < 1$, net investment is negative ($I_t/K_t < \delta$) as firms allow capital to depreciate.



Worked Example 4: Tax-Adjusted User Cost, Desired Capital, and Tobin's q

Suppose an Australian logistics firm faces a real interest rate $r = 0.04$, depreciation rate $\delta = 0.08$, relative capital price $p_K = 1$, corporate tax rate $\tau = 0.30$, depreciation allowance present value $z = 0.80$, and zero tax credit ($k_{ITC} = 0$). Expected output is $Y = 500$ million AUD with capital elasticity $\alpha = 0.36$.

1. **Tax-adjusted user cost (u_K^{tax}):**

$$u_K^{\text{tax}} = \frac{(0.04 + 0.08)(1 - 0.30 \times 0.80)}{1 - 0.30} = \frac{0.12 \times 0.76}{0.70} = \frac{0.0912}{0.70} = 0.1303 \text{ (or } 13.03\%)$$

2. **Desired capital stock (K^*):**

$$K^* = \frac{\alpha Y}{u_K^{\text{tax}}} = \frac{0.36 \times 500}{0.1303} = \frac{180}{0.1303} = 1{,}381.4 \text{ million AUD}$$

3. **Effect of Commonwealth instant asset write-off (z rises from 0.80 to 1.00):** The user cost drops to $u_K^{\text{new}} = \frac{0.12 \times 0.70}{0.70} = 0.1200$ (12.00%, completely eliminating corporate tax distortion on marginal investment), lifting desired capital to $K^{*'} = 180/0.1200 = 1{,}500.0$ million AUD.
4. **Tobin's q investment rate:** Separately, suppose the firm's current capital replacement value is $1{,}000$ million AUD, its stock-market plus debt valuation is $V_t = 1{,}240$ million AUD (so $q_t = 1.24$), and adjustment cost parameter is $\chi = 4.0$. Gross investment rate is:

$$\frac{I_t}{K_t} = 0.08 + \frac{1.24 - 1.00}{4.0} = 0.08 + 0.06 = 0.14 \text{ (or } 14.0\%)$$

Thus gross investment equals 140 million AUD (80 million replacement plus 60 million net expansion).

Chapter 5: Labour Market Microfoundations & Search-and-Matching

5.1 Real Wage Rigidity and Efficiency Wages

In a Walrasian spot labour market, the real wage (w) clears supply and demand ($MPL = w$), leaving zero involuntary unemployment. Real-world labour markets exhibit persistent structural unemployment even at potential output ($u_n > 0$) due to institutional rigidities (such as Australia's Fair Work Commission National Minimum Wage and Modern Awards, and enterprise bargaining) as well as asymmetric-information **efficiency wage** mechanisms:

- **Shirking Model (Shapiro & Stiglitz, 1984):** Because monitoring effort is costly, firms pay an above-market-clearing real wage to raise the penalty of dismissal, creating equilibrium involuntary unemployment as a worker-discipline device.
- **Turnover and Adverse Selection Models:** Higher wages reduce costly quitting/retraining turnover and attract higher-ability applicants.

5.2 The Diamond-Mortensen-Pissarides (DMP) Search-and-Matching Model

Modern macroeconomics explains equilibrium unemployment and vacancy dynamics through the Nobel Prize-winning **Diamond-Mortensen-Pissarides (DMP)** search-and-matching framework. Job creation and job destruction occur simultaneously: unemployed workers (U) search for jobs while firms

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post costly vacancies (V). New matches per period (M) are governed by a constant-returns-to-scale Cobb-Douglas **matching function**:

$$M(U,V) = \mu U^{\eta} V^{1-\eta}, \qquad 0 < \eta < 1$$

Where: M is the flow of new hires; $\mu > 0$ measures matching efficiency (e.g., effectiveness of digital job boards and Workforce Australia employment services); U is the stock of unemployed searchers; V is the stock of job vacancies; and η is the elasticity of matches with respect to unemployment.

Define **labour market tightness** as the vacancy-to-unemployment ratio $\theta = V/U$. Dividing M by U and V respectively yields the **job-finding rate** $f(\theta)$ (probability an unemployed worker finds a job) and the **vacancy-filling rate** $q(\theta)$ (probability a firm fills an open vacancy):

$$f(\theta) = \frac{M}{U} = \mu\theta^{1-\eta}, \qquad q(\theta) = \frac{M}{V} = \mu\theta^{-\eta} = \frac{f(\theta)}{\theta}$$

Where: $\theta = V/U$ is labour market tightness; $f'(\theta) > 0$ (tighter markets help job seekers find work faster); and $q'(\theta) < 0$ (tighter markets create congestion for hiring firms, lowering vacancy-filling probability).

Steady-State Unemployment and the Beveridge Curve

Suppose employed workers ($E = L - U$) lose their jobs at an exogenous separation rate $s \in (0,1)$. In steady state, flows into unemployment ($s(L-U)$) must equal flows out of unemployment ($f(\theta)U$). Dividing by the labour force L gives the steady-state unemployment rate ($u^* = U/L$), tracing out the downward-sloping **Beveridge Curve** in (u,v) space:

$$s(1-u^*) = f(\theta)u^* \implies u^* = \frac{s}{s+f(\theta)} = \frac{s}{s+\mu\theta^{1-\eta}}$$

Where: u^* is the steady-state (natural) unemployment rate; s is the job separation rate; and $f(\theta)$ is the job-finding rate. Movements along the Beveridge curve reflect cyclical shifts in aggregate demand and labour market tightness (θ), whereas outward shifts in the Beveridge curve reflect higher separation rates (s) or lower matching efficiency (μ, structural skill or geographic mismatch).

Wage Nash Bargaining and Job Creation

Once a worker and firm match, they share the match surplus via generalised Nash bargaining, yielding a wage equation combining worker productivity (y), hiring cost per vacancy (c), tightness (θ), outside unemployment income/utility (b), and worker bargaining power ($\beta_N \in (0,1)$):

$$w = \beta_N(y + c\theta) + (1-\beta_N)b$$

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Where: w is the bargained real wage; y is match output (MPL); c is the flow cost of maintaining an open vacancy; b captures unemployment benefits and value of leisure; and β_N is worker bargaining weight. Under free entry of vacancies, firms post vacancies until expected discounted profits equal zero, equating expected hiring cost $c/q(\theta)$ to the firm's share of match surplus $(y - w)/(r + s)$.

Worked Example 5: DMP Steady-State Tightness, Job-Finding, and Unemployment

Consider a labour market calibrated to monthly flows with matching efficiency $\mu = 0.60$, matching elasticity $\eta = 0.50$, monthly job separation rate $s = 0.02$ (2% per month), unemployed pool $U = 600{,}000$, and open vacancies $V = 450{,}000$.

1. **Labour market tightness (θ):**

$$\theta = \frac{V}{U} = \frac{450{,}000}{600{,}000} = 0.75$$

2. **Job-finding rate ($f(\theta)$) and vacancy-filling rate ($q(\theta)$):**

$$f(0.75) = 0.60 \times (0.75)^{0.50} = 0.60 \times 0.8660 = 0.5196 \text{ (or } 51.96\% \text{ per month)}$$

$$q(0.75) = 0.60 \times (0.75)^{-0.50} = \frac{0.60}{0.8660} = 0.6928 \text{ (or } 69.28\% \text{ per month)}$$

Average unemployment duration is $1/f(\theta) = 1/0.5196 \approx 1.92$ months.

3. **Steady-state unemployment rate (u^*):**

$$u^* = \frac{s}{s + f(\theta)} = \frac{0.02}{0.02 + 0.5196} = \frac{0.02}{0.5396} = 0.0371 \text{ (or } 3.71\%)$$

4. **Effect of skill mismatch reducing matching efficiency μ from 0.60 to 0.45 (at $\theta = 0.75$):** The job-finding rate falls to $f'(0.75) = 0.45 \times 0.8660 = 0.3897$, pushing steady-state unemployment up to $u^{*'} = \frac{0.02}{0.02+0.3897} = 0.0488$ (4.88%)—an outward shift of the Beveridge curve by 1.17 percentage points.

Labour Market Paradigm	Source of Unemployment	Wage Determination	Policy Implications
Classical Spot Market	Voluntary leisure / frictionless clearing ($u = 0$)	Market clearing ($MPL = w$)	Supply-side tax/work-incentive policies
Efficiency Wage / Award Rigidity	Above-clearing real wage floor ($w > w^*$)	Firms set w to deter shirking, or Awards set floor	Moderate minimum-wage growth; monitoring tech

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Labour Market Paradigm	Source of Unemployment	Wage Determination	Policy Implications
DMP Search-and-Matching	Search frictions, mismatch (μ), separations (s)	Nash bargaining between matched worker & firm	Retraining, job-matching services ($\mu \uparrow$), balanced b

Chapter 4: Microfoundations of Investment & Financial Frictions

4.1 Tobin's q Theory of Corporate Capital Accumulation

While neoclassical investment theory assumes frictionless capital rental markets, modern corporate investment decisions involve substantial adjustment costs. James Tobin formulated the q theory of investment, linking physical capital expenditure to financial asset market valuations:

$$q = \frac{\text{Market Value of Installed Capital}}{\text{Replacement Cost of Installed Capital}} = \frac{V_t}{P_{K,t} K_t}$$

Where:

- V_t is the equity and debt valuation placed on the firm's capital stock by financial markets (such as the Australian Securities Exchange, ASX).
- $P_{K,t}$ is the current purchase price of uninstalled newly produced physical capital goods.
- K_t is the aggregate physical capital stock of the firm.

When financial markets value installed capital above its physical replacement cost ($q > 1$), firms can issue shares or borrow at attractive terms to purchase additional physical equipment, generating positive net capital accumulation:

$$\frac{I_t}{K_t} = \delta + \phi(q_t - 1)$$

Where δ is the physical depreciation rate, and $\phi > 0$ governs the speed of investment response inversely related to internal installation and capital adjustment costs. Conversely, when $q < 1$, the market values capital below its replacement cost; firms find it cheaper to acquire existing assets via mergers rather than invest in new physical capital, leading to capital decumulation.

4.2 Credit Market Imperfections & The Financial Accelerator

The Modigliani-Miller theorem asserts that corporate investment is independent of financial structure in frictionless markets. In reality, asymmetric information between borrowers and lenders generates agency costs, leading to an external finance premium (EFP):

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$$\text{EFP} = R_{\text{external}} - R_{\text{internal}} = \psi(N_t/A_t), \quad \psi' < 0$$

Where N_t is the firm's net worth (equity and collateral value), and A_t represents total assets. When macroeconomic downturns erode balance sheet values ($N_t \downarrow$), the agency premium rises sharply (EFP $\uparrow$), increasing borrowing costs for healthy firms, choking off business investment, and amplifying the initial economic contraction—a propagation mechanism known as the Bernanke-Gertler-Gilchrist financial accelerator.

4.3 Summary Table: Microeconomic Foundations in Modern Macroeconomic Models

Macroeconomic Block	Key Microeconomic Theory	Core Mechanism	Key Optimising Equation
Household Consumption	Euler Equation / PIH	Intertemporal utility smoothing	$u'(C_t) = \beta(1 + r_t)E_t[u'(C_{t+1})]$
Firm Investment	Tobin's q & Adjustment Costs	Forward-looking asset valuation	$\frac{I_t}{K_t} = \delta + \phi(q_t - 1)$
Financial Intermediation	Financial Accelerator	Collateral constraints & agency costs	$\text{EFP} = \psi(\text{Net Worth/Assets})$
Labour Supply & Search	DMP Search & Matching	Frictional matching & Nash wage bargaining	$\theta = V/U, \quad M = m(U, V)$

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