DegreeLive

Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Intermediate macroeconomics

Course Code: ECC2010 • Module 1 Notes

Module 1: Long-Run Economic Growth: Solow-Swan Model & Endogenous Growth

Chapter 1: Aggregate Production, Factor Markets & National Output

1.1 The Long-Run Horizon and Cross-Country Living Standards

Intermediate macroeconomics separates economic fluctuations into two distinct horizons. In the short run, nominal rigidities in prices and wages cause aggregate demand to govern output, capacity utilisation, and cyclical unemployment. Over horizons spanning decades, however, prices adjust, markets clear, and an economy's material standard of living is determined by its productive capacity—namely, the stocks of physical and human capital, the size and skill of the labour force, and the state of technology and institutional efficiency. Small differences in annual per-capita growth rates compound dramatically over time: an economy growing at 2.5% per person annually doubles real income per capita roughly every 28 years, whereas an economy growing at 1.0% requires 70 years to achieve the same gain.

In Australia, real gross domestic product (GDP) per capita measured by the Australian Bureau of Statistics (ABS) in chain volume terms expanded more than threefold between 1960 and 2025. Understanding why sustained growth occurred—and why Australian multifactor productivity (MFP) decelerated during the 2010s and early 2020s—requires formal growth models rooted in neoclassical production theory and modern endogenous growth frameworks.

1.2 Neoclassical Aggregate Production Functions

Consider a closed aggregate economy in which firms combine physical capital (K), labour (L), and an index of total factor productivity or technology (A) to produce a single homogeneous final good (Y) used for both consumption and investment:

$$Y_t = F(K_t, L_t; A_t)$$

Where: Y_t is real aggregate output (GDP) in period t; K_t is the aggregate stock of physical capital (structures, machinery, equipment, and software); L_t is aggregate labour input (hours worked or employed workers); and A_t is total factor productivity (TFP), capturing the efficiency with which inputs are transformed into output.

A standard neoclassical production function satisfies four canonical properties:

1. **Constant Returns to Scale (Homogeneity of Degree One):** Scaling all rival inputs by any positive constant $\lambda > 0$ scales output by λ, so $F(\lambda K, \lambda L; A) = \lambda F(K, L; A)$.
2. **Positive and Diminishing Marginal Products:** Each additional unit of capital or labour raises output, but at a decreasing rate when the other input is held fixed: $\partial F/\partial K > 0$, $\partial^2 F/\partial K^2 < 0$, $\partial F/\partial L > 0$, and $\partial^2 F/\partial L^2 < 0$.
3. **Inada Conditions:** The marginal product of each input approaches infinity as that input approaches zero ($\lim_{K \to 0} F_K = \infty$) and approaches zero as that input grows without bound ($\lim_{K \to \infty} F_K = 0$). These boundary conditions guarantee an interior steady state.
4. **Essentiality:** Positive output requires strictly positive quantities of both capital and labour ($F(0, L) = F(K, 0) = 0$).

Intensive (Per-Worker) Form and the Cobb-Douglas Specification

Because F exhibits constant returns to scale, setting $\lambda = 1/L$ converts aggregate output into output per worker ($y = Y/L$) as a function of capital per worker ($k = K/L$), known as the capital-labour ratio:

$$y = \frac{Y}{L} = F\left(\frac{K}{L}, 1; A\right) = f(k)$$

degreelive

Where: $y = Y/L$ is labour productivity (output per worker); $k = K/L$ is capital intensity (capital per worker); and $f(k)$ is the intensive production function satisfying $f(0) = 0, f'(k) > 0$, and $f''(k) < 0$.

The benchmark parametric form in empirical macroeconomics and ABS growth accounting is the Cobb-Douglas production function:

$$Y = AK^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1$$

Where: α is the output elasticity of capital (empirically near 0.33 to 0.40 in Australia depending on the treatment of natural resource rents and housing); and $(1-\alpha)$ is the output elasticity of labour. Dividing both sides by L yields the intensive Cobb-Douglas form:

$$y = Ak^{\alpha}$$

Where: y is output per worker; A is Hicks-neutral total factor productivity; k is capital per worker; and α governs the curvature of diminishing returns to capital deepening.

1.3 Competitive Factor Markets and Income Distribution

Under perfect competition, representative firms maximise real profit $\Pi = F(K,L) - RK - wL$, where R is the real rental price of capital and w is the real wage rate. First-order profit maximisation equates factor prices to their marginal products:

$$MPK = \frac{\partial Y}{\partial K} = f'(k) = \alpha A k^{\alpha-1} = R = r + \delta$$

$$MPL = \frac{\partial Y}{\partial L} = f(k) - kf'(k) = (1-\alpha)Ak^{\alpha} = w$$

Where: MPK is the marginal product of capital; MPL is the marginal product of labour; R is the user cost (real rental rate) of capital; r is the real interest rate; δ is the capital depreciation rate; and w is the real wage. By Euler's theorem for homogeneous functions of degree one, total output is exhausted by factor payments: $Y = MPK \cdot K + MPL \cdot L$. Consequently, capital's share of national income is $RK/Y = \alpha$ and labour's share is $wL/Y = 1-\alpha$.

Worked Example 1: Factor Shares, Real Wages, and Capital Returns

Suppose the Australian economy is calibrated with a Cobb-Douglas production function $Y = AK^{0.35}L^{0.65}$, aggregate capital stock K of A$3,200 billion, labour force L of 12.5 million workers, TFP parameter A scaled such that aggregate GDP Y is A$1,280 billion (implying a realistic capital-output ratio $K/Y = 2.50$), and annual depreciation rate $\delta = 0.05$.

1. **Capital-labour ratio (k) and output per worker (y):**

$$k = \frac{3{,}200{,}000 \text{ million}}{12.5 \text{ million}} = 256{,}000 \text{ AUD/worker}, \qquad y = \frac{1{,}280{,}000 \text{ million}}{12.5 \text{ million}} = 102{,}400 \text{ AUD/worker}$$

2. **Calibrating TFP (A):** Measuring k and y in thousands of AUD ($k = 256, y = 102.4$), $256^{0.35} = e^{0.35\ln(256)} = e^{1.9408} \approx 6.964$. Thus $A = 102.4/6.964 \approx 14.70$.

DegreeLive

3. **Real rental rate (R) and real interest rate (r):** Using $MPK = \alpha(Y/K)$ directly:

$$MPK = 0.35 \times \frac{1{,}280}{3{,}200} = \frac{0.35}{2.50} = 0.140 \text{ (or } 14.0\%)$$

   Subtracting depreciation $\delta = 0.05$ yields the competitive real interest rate $r = MPK - \delta = 0.140 - 0.050 = 0.090$ (9.0% gross pre-tax return on physical capital).

4. **Real wage (w) and income shares:** Labour income per worker is $w = (1-\alpha)y$, giving $0.65 \times 102{,}400 =$ A$66,560 (A$66,560). Total labour compensation is $0.65 \times 1{,}280$ bn $= 832$ bn AUD (65% of GDP), while total gross capital income is $0.35 \times 1{,}280$ bn $= 448$ bn AUD (35% of GDP).

Chapter 2: The Solow-Swan Capital Accumulation Model

2.1 Saving, Investment, and the Law of Motion for Capital

Developed independently by Robert Solow and Trevor Swan (an Australian economist at the Australian National University) in 1956, the Solow-Swan model explains how capital accumulation, labour force growth, and technological progress interact to determine an economy's long-run steady state and transitional growth path. In a closed economy with no government sector (or where s represents the combined national saving rate), national saving equals aggregate investment ($S = I = sY$), where $s \in (0,1)$ is an exogenous, constant saving rate.

The aggregate capital stock evolves according to gross investment minus physical depreciation:

$$\dot{K}_t = \frac{dK_t}{dt} = sY_t - \delta K_t$$

Where: $\dot{K}_t$ is the instantaneous change in the aggregate capital stock; s is the national saving rate; Y_t is aggregate output; and $\delta \in (0,1)$ is the annual depreciation rate of capital. Simultaneously, the labour force grows at a constant exogenous rate $n = \dot{L}_t/L_t$.

To derive the dynamics of capital per worker $k_t = K_t/L_t$, take the logarithmic derivative with respect to time:

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = \frac{sY_t - \delta K_t}{K_t} - n = \frac{sf(k_t)}{k_t} - (n+\delta)$$

Multiplying both sides by k_t produces the fundamental differential equation of the Solow-Swan model:

$$\dot{k}_t = sf(k_t) - (n+\delta)k_t$$

Where: $\dot{k}_t$ is the rate of capital deepening (change in capital per worker); $sf(k_t)$ is actual investment per worker; and $(n+\delta)k_t$ is break-even investment per worker, defined as the investment required to equip n new workers with k_t units of capital each and replace δk_t units of worn-out capital. Whenever actual investment exceeds break-even investment ($sf(k_t) > (n+\delta)k_t$), capital per worker rises ($\dot{k}_t > 0$); whenever actual investment falls short, capital per worker declines ($\dot{k}_t < 0$).

DegreeLive

2.2 Steady-State Equilibrium and Closed-Form Solutions

A steady state is a long-run equilibrium in which capital per worker remains constant ($\dot{k} = 0$). Setting $\dot{k} = 0$ defines the steady-state capital-labour ratio k^*:

$$sf(k^*) = (n + \delta)k^*$$

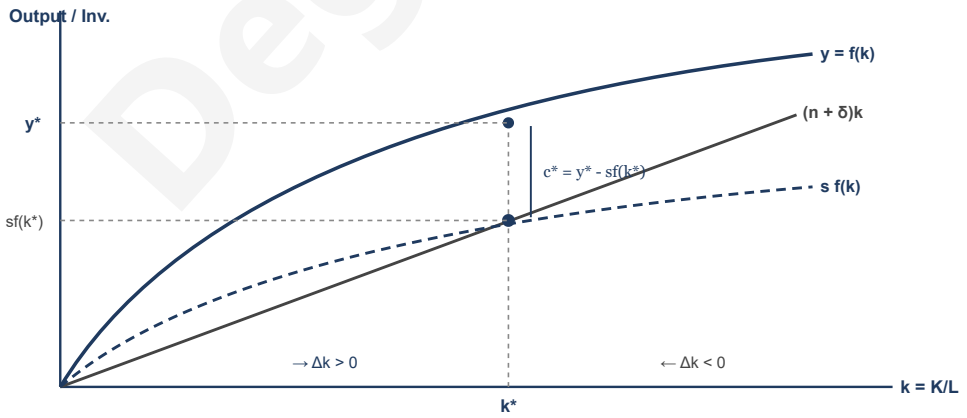
Where: k^* is the steady-state level of capital per worker; $sf(k^*)$ is steady-state saving/investment per worker; and $(n + \delta)k^*$ is steady-state break-even investment per worker. For the Cobb-Douglas production function $f(k) = Ak^{\alpha}$, substituting into the steady-state condition yields $sA(k^*)^{\alpha} = (n + \delta)k^*$, which solves explicitly for k^*, steady-state output per worker y^*, and steady-state consumption per worker c^*:

$$k^* = \left(\frac{sA}{n + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = A(k^*)^{\alpha} = A^{\frac{1}{1-\alpha}} \left(\frac{s}{n + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$c^* = (1 - s)y^* = (1 - s)A^{\frac{1}{1-\alpha}} \left(\frac{s}{n + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Where: k^* is steady-state capital per worker; y^* is steady-state output per worker; c^* is steady-state consumption per worker; s is the national saving rate; A is total factor productivity; n is the labour force growth rate; δ is the depreciation rate; and α is capital's share of income.



2.3 Comparative Statics: Saving, Population Growth, and Level vs. Growth Effects

A fundamental insight of the Solow-Swan model is the distinction between *level effects* and *growth-rate effects*. Consider an economy initially at steady state k_0^* that experiences a permanent increase in its national saving rate from s_1 to $s_2 > s_1$ (for example, through Australia's gradual rise in the Superannuation Guarantee from 9.5% to 12%).

DegreeLive

- **Immediate Transitional Impact:** The actual investment schedule $sf(k)$ shifts upward. At k_0^*, actual investment now exceeds break-even investment ($s_2 f(k_0^*) > (n+\delta)k_0^*$), so $\dot{k} > 0$. Both capital per worker and output per worker begin growing faster than their steady-state rates.
- **Diminishing Transitional Growth:** As k rises, diminishing returns to capital reduce the marginal boost to output, while break-even investment $(n+\delta)k$ rises linearly. The gap $s_2 f(k) - (n+\delta)k$ narrows until the economy reaches a higher steady state $k_1^* > k_0^*$ and $y_1^* > y_0^*$.
- **Long-Run Outcome:** Once at k_1^*, per-worker growth ceases ($\dot{k} = 0, \dot{y}/y = 0$ without exogenous technological progress). Thus, a higher saving rate permanently raises the *level* of steady-state income per worker (y^*), but only *temporarily* raises the growth rate during the transition.

Conversely, an increase in the labour force growth rate from n_1 to $n_2 > n_1$ steepens the break-even line $(n+\delta)k$. Because more investment is required simply to equip newly entering workers ("capital widening"), steady-state capital per worker k^* and output per worker y^* fall, although total aggregate GDP ($Y = y \cdot L$) grows faster in the long run at rate n_2.

Worked Example 2: Steady-State Calculation and Saving Rate Expansion

Suppose an economy without technological progress has $y = Ak^{1/3}$ with $A = 3$, initial national saving rate $s_1 = 0.20$, labour growth $n = 0.02$, and depreciation $\delta = 0.08$.

1. **Initial steady state (k_1^*, y_1^*, c_1^*):** Here $n + \delta = 0.02 + 0.08 = 0.10$ and $1 - \alpha = 2/3$, so $\frac{1}{1-\alpha} = 1.5$.

$$k_1^* = \left(\frac{0.20 \times 3}{0.10}\right)^{1.5} = 6^{1.5} = 6 \sqrt{6} \approx 14.697$$

$$y_1^* = 3 \times (14.697)^{1/3} = 3 \times 6^{0.5} \approx 7.348$$

$$c_1^* = (1 - 0.20) \times 7.348 = 5.879$$

2. **Permanent increase in saving to $s_2 = 0.27$:**

$$k_2^* = \left(\frac{0.27 \times 3}{0.10}\right)^{1.5} = (8.1)^{1.5} = 8.1 \times \sqrt{8.1} = 8.1 \times 2.846 = 23.053$$

$$y_2^* = 3 \times (8.1)^{0.5} = 3 \times 2.846 = 8.538$$

$$c_2^* = (1 - 0.27) \times 8.538 = 6.233$$

3. **Immediate transitional growth rate of k at k_1^*:** Immediately after s jumps to 0.27, actual saving per unit of capital rises, so:

DegreeLive

$$\frac{\dot{k}}{k} = \frac{s_2 y_1^*}{k_1^*} - (n+\delta) = \frac{0.27 \times 7.348}{14.697} - 0.10 = 0.135 - 0.10 = 0.035 \text{ (or } 3.5\%)$$

Because $\dot{y}/y = \alpha(\dot{k}/k)$, output per worker immediately accelerates from 0% to $\frac{1}{3} \times 3.5\% \approx 1.17\%$ per year before gradually decaying back toward zero as k approaches $k_2^* = 23.053$.

Chapter 3: The Golden Rule Level of Capital & Dynamic Efficiency

3.1 Maximising Steady-State Consumption per Worker

Higher saving always raises steady-state output per worker y^*, but it does not necessarily raise steady-state consumption per worker c^*, because a larger fraction of output must be diverted to maintain a larger depreciating capital stock. Society's welfare in steady state depends on consumption c^*, not output y^*. Substituting $sf(k^*) = (n+\delta)k^*$ into the consumption identity yields steady-state consumption as a function of the chosen steady-state capital stock k^*:

$$c^*(k^*) = y^* - i^* = f(k^*) - (n+\delta)k^*$$

Where: $c^*(k^*)$ is steady-state consumption per worker; $f(k^*)$ is steady-state output per worker; and $(n+\delta)k^*$ is steady-state break-even investment per worker.

The **Golden Rule level of capital** (k_{gold}^*) is the steady-state capital-labour ratio that maximises $c^*(k^*)$. Differentiating $c^*(k^*)$ with respect to k^* and setting the derivative to zero gives the Golden Rule condition:

$$\frac{dc^*}{dk^*} = f'(k_{gold}^*) - (n+\delta) = 0 \implies MPK(k_{gold}^*) = n + \delta$$

Where: $f'(k_{gold}^*) = MPK(k_{gold}^*)$ is the marginal product of capital at the Golden Rule; n is the labour force growth rate; and δ is the depreciation rate. Equivalently, subtracting δ from both sides gives the net marginal product of capital condition $r_{gold}^* = MPK - \delta = n$: at the Golden Rule, the net real return on capital equals the economy's steady-state growth rate (or $n + g_A$ when technology grows at g_A).

For a Cobb-Douglas production function $y = Ak^\alpha$, multiplying $f'(k_{gold}^*) = \alpha y/k$ by k/y shows that the Golden Rule saving rate (s_{gold}) equals capital's income share α:

$$s_{gold} = \frac{(n+\delta)k_{gold}^*}{f(k_{gold}^*)} = \frac{f'(k_{gold}^*)k_{gold}^*}{f(k_{gold}^*)} = \alpha$$

Where: s_{gold} is the saving rate that implements the Golden Rule steady state; and α is the output elasticity of capital.

3.2 Dynamic Efficiency vs. Dynamic Inefficiency

Comparing an economy's actual steady state k^* to k_{gold}^* reveals two distinct policy regimes:

- **Under-accumulation ($k^* < k_{gold}^*$, or $MPK > n + \delta$, equivalently $s < \alpha$):** The economy is *dynamically efficient*. Raising the saving rate s toward s_{gold} raises long-run consumption c^*, but imposes a short-run sacrifice: current generations must cut consumption immediately to build up capital, enjoying higher consumption only after capital accumulates sufficiently. Intertemporal welfare trade-offs depend on the social discount rate.

DegreeLive

- **Over-accumulation ($k^* > k^*_{gold}$, or $MPK < n + \delta$, equivalently $s > \alpha$):** The economy is *dynamically inefficient*. So much output is absorbed replacing depreciating capital that lowering the saving rate s toward s_{gold} increases consumption both *immediately* and in *every future period*—a strict Pareto improvement. Empirical tests for OECD economies (including Australia, where $MPK - \delta$ exceeds GDP growth) consistently indicate dynamic efficiency ($k^* < k^*_{gold}$).

Worked Example 3: Solving the Golden Rule and Evaluating Dynamic Efficiency

Consider an economy with Cobb-Douglas production $y = 4k^{0.30}$, labour growth $n = 0.015$, depreciation $\delta = 0.065$, and current saving rate $s = 0.24$.

1. **Golden Rule saving rate, capital, output, and consumption:** Here $n + \delta = 0.08$ and $\alpha = 0.30$, so $s_{gold} = \alpha = 0.30$. Solving $0.30 \times 4(k^*_{gold})^{-0.70} = 0.08$ gives:

$$(k^*_{gold})^{0.70} = \frac{1.20}{0.08} = 15 \implies k^*_{gold} = 15^{1/0.70} = 15^{1.4286} \approx 47.90$$

$$y^*_{gold} = 4 \times (47.90)^{0.30} = 4 \times 3.191 = 12.765$$

$$c^*_{gold} = (1 - 0.30) \times 12.765 = 8.936$$

2. **Current steady state at $s = 0.24$:**

$$k^* = \left(\frac{0.24 \times 4}{0.08}\right)^{1.4286} = 12^{1.4286} \approx 34.81$$

$$y^* = 4 \times (34.81)^{0.30} = 4 \times 2.900 = 11.600$$

$$c^* = (1 - 0.24) \times 11.600 = 8.816$$

3. **Dynamic efficiency test:** At $s = 0.24$, $k^* = 34.81 < k^*_{gold} = 47.90$, and $MPK = 0.30 \times (11.600/34.81) = 0.100 > n + \delta = 0.080$. The economy is dynamically efficient: achieving $c^*_{gold} = 8.936$ would require raising s from 24% to 30%, temporarily reducing consumption from 8.816 to $(1 - 0.30) \times 11.600 = 8.120$ during the transition before eventually exceeding 8.816.

Chapter 4: Technological Progress, TFP & Growth Accounting

4.1 Harrod-Neutral Technological Progress and Effective Labour

In the baseline Solow-Swan model without technological change, steady-state growth in output per worker is zero—contradicting over two centuries of sustained growth in Australia and advanced economies. Introducing labour-augmenting (Harrod-neutral) technological progress resolves this puzzle while preserving a constant capital-output ratio and constant factor shares along a balanced growth path:

DegreeLive

$$Y_t = F(K_t, A_t L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

Where: A_t is the labour-augmenting technology index growing exogenously at rate $g_A = \dot{A}_t / A_t$; and $A_t L_t$ represents the stock of *effective labour* (or efficiency units of labour), growing at rate $n + g_A$.

Define output per effective worker $\widetilde{y}_t = Y_t/(A_t L_t)$ and capital per effective worker $\widetilde{k}_t = K_t/(A_t L_t)$. Dividing the production function by $A_t L_t$ gives $\widetilde{y}_t = \widetilde{k}_t^{\alpha}$, and taking logarithmic derivatives yields the augmented capital transition equation:

$$\dot{\widetilde{k}}_t = s\widetilde{k}_t^{\alpha} - (n + g_A + \delta)\widetilde{k}_t$$

Where: $\dot{\widetilde{k}}_t$ is the rate of change of capital per effective worker; s is the saving rate; n is labour growth; g_A is the rate of labour-augmenting technological progress; and δ is depreciation. At the steady state $\dot{\widetilde{k}} = 0$:

$$\widetilde{k}^* = \left(\frac{s}{n + g_A + \delta}\right)^{\frac{1}{1-\alpha}}, \qquad \widetilde{y}^* = \left(\frac{s}{n + g_A + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Where: $\widetilde{k}^*$ is steady-state capital per effective worker; and $\widetilde{y}^*$ is steady-state output per effective worker.

4.2 Balanced Growth Path Properties (Kaldor Stylised Facts)

Along the steady-state balanced growth path, $\widetilde{k}^*$ and $\widetilde{y}^*$ are constant, which pins down the long-run growth rates of all macroeconomic aggregates:

Macroeconomic Variable	Symbol	Steady-State Growth Rate	Economic Interpretation
Capital per effective worker	$\widetilde{k} = K/(AL)$	0	Constant ratio along balanced growth path
Output per effective worker	$\widetilde{y} = Y/(AL)$	0	Constant ratio along balanced growth path
Capital per worker	$k = K/L = A\widetilde{k}$	g_A	Capital deepening matches technological progress
Output per worker (labour productivity)	$y = Y/L = A\widetilde{y}$	g_A	Sole driver of sustained living-standard growth
Real wage rate	$w = (1-\alpha)y$	g_A	Real wages track labour productivity
Aggregate capital & GDP	K and Y	$n + g_A$	Extensive growth plus productivity growth
Real interest rate & capital-output ratio	$r = MPK - \delta$, K/Y	0	Factor returns and K/Y remain trendless

4.3 Growth Accounting and the Solow Residual

To measure the empirical contributions of capital accumulation, labour expansion, and technological innovation to observed GDP growth, Robert Solow (1957) developed the growth accounting decomposition. Starting from $Y_t = A_t F(K_t, L_t)$ and taking total differentials with respect to time gives:

DegreeLive

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

Where: $\Delta Y/Y$ is the growth rate of real GDP; $\Delta K/K$ is the growth rate of physical capital services; $\Delta L/L$ is the growth rate of total hours worked (or quality-adjusted labour); α and $(1-\alpha)$ are the competitive income shares of capital and labour; and $\Delta A/A$ is the **Solow residual** (referred to by the ABS as multifactor productivity, MFP), calculated as the portion of output growth unexplained by measured input growth:

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

Subtracting $\Delta L/L$ from both sides converts output growth accounting into a decomposition of **labour productivity growth** ($\Delta y/y = \Delta Y/Y - \Delta L/L$):

$$\frac{\Delta y}{y} = \frac{\Delta A}{A} + \alpha\frac{\Delta k}{k}$$

Where: $\Delta y/y$ is labour productivity growth (output per hour worked); $\Delta A/A$ is MFP growth; and $\alpha(\Delta k/k)$ is the contribution of *capital deepening* (growth in capital services per hour worked weighted by capital's income share). Note carefully the algebraic relationship between Hicks-neutral TFP growth ($\Delta A/A$) and Harrod-neutral labour-augmenting growth (g_A): for Cobb-Douglas, $A_{\text{Hicks}} = A_{\text{Harrod}}^{1-\alpha}$, so along a balanced growth path where $\Delta k/k = g_A$, $\Delta A/A = (1-\alpha)g_A$, and labour productivity grows at $\Delta y/y = (1-\alpha)g_A + \alpha g_A = g_A$.

Worked Example 4: ABS-Style Growth Accounting Decomposition

Suppose over a five-year productivity cycle in Australia's market sector, real output grows at 3.10% per year, quality-adjusted hours worked grow at 1.40% per year, and capital services grow at 3.80% per year, with capital income share $\alpha = 0.36$ and labour income share $1-\alpha = 0.64$.

1. **Input contributions to GDP growth:**
   - Capital contribution: $\alpha(\Delta K/K) = 0.36 \times 3.80\% = 1.368\%$ per year.
   - Labour contribution: $(1-\alpha)(\Delta L/L) = 0.64 \times 1.40\% = 0.896\%$ per year.
   - Combined factor input growth: $1.368\% + 0.896\% = 2.264\%$ per year.
2. **Multifactor productivity (Solow residual):**

$$\frac{\Delta A}{A} = 3.100\% - 2.264\% = 0.836\% \text{ per year}$$

3. **Labour productivity decomposition:** Labour productivity growth is $\Delta y/y = 3.100\% - 1.400\% = 1.700\%$ per year. Capital deepening is $\Delta k/k = 3.800\% - 1.400\% = 2.400\%$, contributing $0.36 \times 2.400\% = 0.864\%$ per year. Thus, $0.864\% + 0.836\% = 1.700\%$: capital deepening accounts for 50.8% of labour productivity growth and MFP accounts for 49.2%.

Degreelive

Chapter 5: Convergence Hypotheses & Endogenous Growth Theory

5.1 Absolute Convergence, Conditional Convergence, and Speed of Adjustment

A central prediction of the Solow-Swan model concerns whether poorer economies catch up to richer economies over time. Because diminishing returns to capital imply a higher marginal product of capital when $\tilde{k}$ is low, economies further below their steady state experience faster transitional growth:

- **Unconditional (Absolute) Convergence (β-convergence):** If all countries share identical structural parameters (s, n, δ, g_A, and α) and access to the same technology A, initially poorer economies (with lower k_0) will grow faster than richer ones and converge to the exact same steady-state income per capita y^*. Empirically, absolute convergence holds strongly among relatively homogeneous groups such as OECD nations or Australian states, but fails across the entire world sample because poorer countries often have lower saving rates, higher population growth, and weaker institutions.
- **Conditional Convergence:** Controlling for differences in s, n, human capital, and institutional quality (A), each country converges toward its *own* country-specific steady state y_i^*. An economy grows faster the further its current output lies below its own steady state.
- **σ-Convergence:** Refers to a decline over time in the cross-sectional dispersion (standard deviation) of log income per capita across economies. β-convergence is necessary but not sufficient for σ-convergence if economies are hit by large idiosyncratic shocks.

Linearising the Solow transition equation $\dot{\tilde{k}} = s\tilde{k}^{\alpha} - (n + g_A + \delta)\tilde{k}$ around $\tilde{k}^*$ yields the **speed of convergence** (λ) governing how rapidly the gap $\ln \tilde{y}_t - \ln \tilde{y}^*$ closes each year:

$$\frac{d \ln \tilde{y}_t}{dt} \approx -\lambda \left(\ln \tilde{y}_t - \ln \tilde{y}^* \right), \qquad \lambda = (1 - \alpha)(n + g_A + \delta)$$

Where: λ is the annual convergence rate; α is capital's output share; n is population growth; g_A is technological progress; and δ is capital depreciation. With standard parameters ($\alpha = 1/3,\ n = 0.01,\ g_A = 0.02,\ \delta = 0.07$), $\lambda = (2/3)(0.10) \approx 0.067$, implying a half-life of $\ln(2)/0.067 \approx 10.4$ years—though empirical estimates around $\lambda \approx 0.02$ suggest broader capital concepts ($\alpha \approx 0.75$ including human capital, as in Mankiw, Romer, and Weil 1992) match cross-country data.

5.2 The AK Model of Endogenous Growth

Because the Solow-Swan model treats g_A as exogenous, it leaves the ultimate engine of long-run per-capita growth unexplained. Endogenous growth theory internalises technological progress and broad capital accumulation into economic decisions. The simplest endogenous growth model eliminates diminishing returns to aggregate capital by setting $\alpha = 1$ in a broad capital composite K (encompassing physical capital, human skills, and disembodied knowledge):

$$Y_t = AK_t$$

Where: Y_t is aggregate output; $A > 0$ is a constant marginal and average product of broad capital; and K_t is the broad capital stock. Substituting $Y_t = AK_t$ into $\dot{K}_t = sY_t - \delta K_t$ and dividing by K_t (assuming constant population $n = 0$ for clarity, or per-capita k under constant returns to K and L jointly if A captures scale externalities) yields a constant, permanent growth rate:

DegreeLive

$$g_Y = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sA - \delta$$

Where: g_Y is the permanent endogenous growth rate; s is the saving/investment rate; A is capital productivity; and δ is the depreciation rate. Unlike the Solow-Swan model, there is no steady-state level k^* toward which growth decays: whenever $sA > \delta$, the economy grows perpetually at rate $sA - \delta$, and a permanent increase in the saving rate s permanently increases the *long-run growth rate*, not merely the level of income.

5.3 Romer's Knowledge-Driven R&D Model and Human Capital

Paul Romer (1990) grounded endogenous growth in the microeconomics of ideas. Unlike physical capital or labour, ideas (blueprints, algorithms, chemical formulas) are **non-rival**: one firm's use of a production blueprint does not prevent other firms from using it simultaneously. Because non-rival ideas can be replicated across an entire economy, production exhibits *increasing returns* to all inputs (capital, labour, and ideas) taken together, requiring imperfect competition (monopoly rents protected by patents) to cover fixed R&D costs.

In a simplified Romer framework, total labour L is allocated between goods production ($L_Y = (1 - s_R)L$) and research and development ($L_A = s_R L$). Final output and new idea creation follow:

$$Y_t = A_t^{\sigma} L_Y, \qquad \dot{A}_t = \theta L_A^{\lambda} A_t^{\phi}$$

Where: $s_R \in (0, 1)$ is the R&D labour share; $\theta > 0$ is research productivity; $\lambda \in (0, 1]$ captures duplication externalities in R&D; and ϕ captures intertemporal knowledge spillovers ("standing on shoulders" when $\phi > 0$, or "fishing out" when $\phi < 0$). In the benchmark Romer case where $\lambda = 1$ and $\phi = 1$, dividing $\dot{A}_t$ by A_t gives a steady-state technological growth rate proportional to research effort: $g_A = \theta s_R L$. In Jones's (1995) semi-endogenous variant ($\phi < 1$), balanced growth requires $g_A = \frac{\lambda n}{1-\phi}$, linking long-run growth to population growth and avoiding counterfactual scale effects where larger populations grow arbitrarily faster.

Complementing R&D models, Robert Lucas (1988) emphasised **human capital accumulation** (H): workers split time between production (u) and education/training ($1 - u$), producing output $Y = AK^{\alpha}(uH)^{1-\alpha}$ and accumulating skills via $\dot{H} = \psi(1 - u)H$. Because human capital augments effective labour without encountering diminishing returns at the aggregate level, education subsidies and higher schooling quality permanently raise growth.

Worked Example 5: AK Policy Growth Impact vs. Solow Transition

Consider an AK economy with $Y = 0.40K$, initial saving rate $s = 0.25$, depreciation $\delta = 0.06$, and initial capital $K_0 = 1{,}000$ billion dollars.

1. **Baseline growth rate:** $g_Y = sA - \delta = (0.25 \times 0.40) - 0.06 = 0.10 - 0.06 = 0.04$ (4.0% per year permanently). Initial output is $Y_0 = 0.40 \times 1{,}000 = 400$ billion dollars; after 10 years, $Y_{10} = 400 \times e^{0.04 \times 10} = 400 \times 1.4918 = 596.7$ billion dollars.
2. **Effect of an R&D / investment tax reform raising s to 0.30:** The permanent growth rate rises to $g_Y' = (0.30 \times 0.40) - 0.06 = 0.12 - 0.06 = 0.06$ (6.0% per year). After 10 years, $Y_{10}' = 400 \times e^{0.06 \times 10} = 400 \times 1.8221 = 728.8$ billion dollars—22.1% higher than baseline, with the gap widening exponentially over time rather than converging to a fixed level ratio as in Solow-Swan.

Chapter 6: Australian Productivity Trends & Growth Policy

6.1 Historical Phases of Australian Productivity Growth

ABS multifactor productivity (MFP) and labour productivity series reveal three distinct eras in modern Australian economic growth, analysed extensively by the Productivity Commission and the Reserve Bank of Australia:

DegreeLive

- **The 1990s Microeconomic Reform Boom (1993–2000):** Following floating of the Australian dollar, tariff reductions, National Competition Policy, enterprise bargaining, and rapid ICT adoption, Australian market-sector MFP grew around 1.8% to 2.1% per year, lifting labour productivity above 3.0% annually.
- **The Mining Investment Boom & Capital Deepening Phase (2003–2013):** Surging Chinese commodity demand pushed Australia's terms of trade to historic highs. Massive capital expenditure in LNG and iron ore projects raised the capital-labour ratio rapidly, sustaining GDP growth, while measured MFP temporarily declined because large capital outlays preceded full resource extraction output.
- **The Post-2016 Productivity Slowdown:** Over the decade leading into the mid-2020s, Australian labour productivity growth slowed toward 0.5%–1.0% per year and MFP growth hovered near zero to 0.3%. Contributing factors include slower technological diffusion from the global frontier, declining business dynamism and startup entry rates, regulatory complexity, and structural shifts toward labour-intensive care and service sectors (healthcare, aged care, NDIS) where measured productivity grows slowly (Baumol's cost disease).

6.2 Synthesis of Growth Models and Structural Policy Levers

Feature	Solow-Swan Model	AK Endogenous Model	Romer R&D Ideas Model
Returns to reproducible capital	Diminishing ($0 < \alpha < 1$)	Constant ($\alpha = 1$)	Diminishing in K, non-rival ideas
Long-run per-capita growth driver	Exogenous g_A only	$sA - \delta$ (saving & efficiency)	R&D allocation $s_R L$ & spillovers
Effect of higher saving rate s	Permanent level effect; temporary growth effect	Permanent growth-rate effect	Level effect unless raising R&D share
Convergence prediction	Conditional convergence	No convergence (divergent paths)	Conditional on technology diffusion
Key Australian policy levers	Superannuation saving, skilled migration (n & H)	Infrastructure, education, tax neutrality	R&D tax incentive, competition, IP law

From an Australian macroeconomic policy standpoint, demographic ageing places downward pressure on labour force participation and hours worked per capita (L/Pop). As the Commonwealth Treasury's *Intergenerational Reports* emphasise, because population ageing acts as a headwind to extensive growth, sustaining Australian living standards over coming decades depends overwhelmingly on raising g_A through competition reform, tertiary education, digital adoption, energy transition investment, and efficient capital allocation.

DegreeLive