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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Intermediate microeconomics

Course Code: ECC2000 • Module 3 Notes

Module 3: Competitive Equilibrium, Monopoly Pricing, Price Discrimination & Oligopoly

Chapter 9: Profit Maximisation, Competitive Firm Supply & Industry Equilibrium

9.1 Profit Maximisation in Output and Input Space

A price-taking competitive firm faces a fixed market output price $p > 0$ and competitive factor prices $(w, r) \gg 0$. In input space, the firm chooses labour and capital (L, K) directly to maximise economic profit:

$$\pi(p, w, r) = \max_{K,L \geq 0} \; p \cdot f(K, L) - wL - rK$$

Where:

- $\pi(p, w, r)$ is the **indirect profit function**, giving maximum attainable economic profit.
- $p \cdot f(K, L)$ is total revenue and $wL + rK$ is total economic factor cost (including opportunity cost of owner-supplied capital).

At an interior maximum, the first-order conditions require equating the Value of Marginal Product ($VMP_i = p \cdot MP_i$) to each factor's price:

$$p \cdot MP_L(K^*, L^*) = w, \qquad p \cdot MP_K(K^*, L^*) = r$$

Where MP_L and MP_K are the marginal physical products of labour and capital. Second-order sufficiency requires decreasing returns to scale locally (a strictly concave production function, $f_{LL} < 0, f_{KK} < 0$, and $f_{LL} f_{KK} - f_{LK}^2 > 0$); under global constant returns to scale, competitive profit is either zero (at $p = \min LAC$) or infinite.

Equivalently, in output space the firm chooses q to maximise $pq - C(w, r, q)$, where $C(w, r, q)$ is the minimum cost function from Module 2. Differentiating $\pi(p, w, r)$ with respect to prices via the Envelope Theorem yields **Hotelling's Lemma**:

$$q^*(p, w, r) = \frac{\partial \pi(p, w, r)}{\partial p}, \qquad L^*(p, w, r) = -\frac{\partial \pi(p, w, r)}{\partial w}, \qquad K^*(p, w, r) = -\frac{\partial \pi(p, w, r)}{\partial r}$$

Where $q^*(p, w, r)$ is the competitive output supply function and L^*, K^* are unconditional (profit-maximising) factor demands. Because $\pi(p, w, r)$ is a maximum of affine functions in (p, w, r), it is convex in (p, w, r), implying $\frac{\partial q^*}{\partial p} = \frac{\partial^2 \pi}{\partial p^2} \geq 0$ (supply curves never slope downward) and $\frac{\partial L^*}{\partial w} = -\frac{\partial^2 \pi}{\partial w^2} \leq 0$ (unconditional factor demands never slope upward).

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9.2 Short-Run Shutdown, Long-Run Exit, and Industry Supply

In the short run, fixed cost $FC = r\bar{K}$ is sunk and cannot be avoided by producing $q = 0$. Therefore, a competitive firm produces where $p = SMC(q)$ (along the rising portion where $SMC' > 0$) provided revenue covers avoidable variable cost ($pq \geq VC(q) \iff p \geq AVC(q)$). Thus, the firm's short-run supply curve is:

$$q_{SR}(p) = \begin{cases} SMC^{-1}(p) & \text{if } p \geq \min AVC(q) \\ 0 & \text{if } p < \min AVC(q) \end{cases}$$

Where $\min AVC(q)$ is the **shutdown price**. If $\min AVC < p < \min SAC$, the firm operates at a negative accounting/economic profit, yet loses strictly less than $-FC$ because revenue covers variable costs and contributes $pq - VC(q) > 0$ toward fixed obligations. In the long run, all costs are avoidable, so the firm exits whenever $p < \min LAC(q)$, making the long-run firm supply the segment of $LMC(q)$ above $\min LAC(q)$.

In a short-run industry with a fixed number N of identical active firms, market supply is the horizontal sum $Q_S(p) = N \cdot q_{SR}(p)$. In the **long run with free entry and exit**, positive economic profits attract entrants (shifting industry supply right and lowering p) while negative profits induce exit until every active firm earns zero economic profit:

$$p^* = LMC(q^*) = \min LAC(q^*), \qquad N^* = \frac{Q_D(p^*)}{q^*}$$

Where q^* is each firm's efficient scale output and N^* is the equilibrium number of firms. If input prices remain invariant as industry output expands, the long-run industry supply curve is horizontal at $p^* = \min LAC$ (a **constant-cost industry**). If industry expansion bids up scarce specialised inputs (such as skilled mining engineers in Western Australia or irrigated horticulture land in the Murray-Darling Basin), factor prices rise, shifting firm LAC upward and producing an upward-sloping long-run industry supply curve (an **increasing-cost industry**).

9.3 Partial Equilibrium Welfare, Tax Incidence, and Deadweight Loss

In a competitive market without externalities, equilibrium price p^* equates marginal willingness to pay ($P_D(Q)$) to marginal cost ($MC(Q)$), maximising total marshallian surplus $W = CS + PS$. When the Australian Government levies a specific excise tax t per unit (say on fuel or alcohol), a wedge $p_b - p_s = t$ opens between the buyer price p_b and seller price p_s. For linear supply and demand—or locally around equilibrium—the share of the tax burden borne by buyers versus sellers depends solely on the relative magnitudes of price elasticities:

$$\frac{dp_b}{dt} = \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|}, \qquad \frac{dp_s}{dt} = -\frac{|\varepsilon_D|}{\varepsilon_S + |\varepsilon_D|}$$

Where:

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- $|\varepsilon_D|$ is the absolute price elasticity of demand and ε_S is the price elasticity of supply.
- The less elastic side of the market bears the larger share of the statutory tax, irrespective of whether buyers or sellers legally remit the tax to the Australian Taxation Office (ATO).

Reducing equilibrium quantity from Q^* to Q_t creates a **Harberger Deadweight Loss (DWL)** triangle measuring mutually beneficial trades prevented by the tax wedge:

$$DWL \approx \frac{1}{2} t (Q^* - Q_t) = \frac{1}{2} \frac{|\varepsilon_D| \varepsilon_S}{|\varepsilon_D| + \varepsilon_S} \frac{t^2}{p^*} Q^*$$

Where DWL grows quadratically with the tax rate t: doubling a specific tax quadruples its excess efficiency burden.

Worked Example 9.1: Competitive Long-Run Entry Equilibrium and Tax Incidence

Suppose the Australian craft-roasting wholesale market is perfectly competitive with identical firms each having long-run total cost $LTC(q) = 200 + 10q + 2q^2$, and market demand is $Q_D(p) = 1{,}000 - 10p$.

1. **Step 1 — Efficient scale and long-run equilibrium price:** Long-run marginal and average costs are $LMC(q) = 10 + 4q$ and $LAC(q) = \frac{200}{q} + 10 + 2q$. Setting $LMC(q) = LAC(q)$ (or $\frac{dLAC}{dq} = 0$):

$$10 + 4q = \frac{200}{q} + 10 + 2q \implies 2q = \frac{200}{q} \implies q^2 = 100 \implies q^* = 10$$

   Substituting $q^* = 10$ gives $p^* = \min LAC = 10 + 4(10) = 50$ dollars per unit.

2. **Step 2 — Industry output and equilibrium number of firms:** At $p^* = 50$, industry demand is $Q^* = 1{,}000 - 10(50) = 500$ units. Free entry yields $N^* = 500/10 = 50$ firms, each earning zero economic profit ($50(10) - [200 + 10(10) + 2(100)] = 0$).

3. **Step 3 — Short-run specific tax (t = \$14 per unit) with $N = 50$ fixed firms:** Each firm equates $p_s = SMC(q) = 10 + 4q$, so firm supply is $q(p_s) = (p_s - 10)/4$, and short-run industry supply is $Q_S(p_s) = 50 \cdot \frac{p_s - 10}{4} = 12.5 p_s - 125$. Setting $Q_D(p_b) = Q_S(p_b - 14)$:

$$1{,}000 - 10p_b = 12.5(p_b - 14) - 125 = 12.5p_b - 300 \implies 22.5p_b = 1{,}300 \implies p_b = 57.78$$

   Thus buyers pay $p_b = 57.78$ dollars (absorbing \$7.78, or 55.6% of the tax) and sellers receive $p_s = 43.78$ dollars (absorbing \$6.22, or 44.4%). Quantity falls to $Q_t = 1{,}000 - 10(57.78) = 422.22$, generating tax revenue $T = 14 \times 422.22 = 5{,}911.11$ dollars and deadweight loss $DWL = \frac{1}{2}(14)(500 - 422.22) = 544.44$ dollars.

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Chapter 10: Monopoly Power, Markup Pricing, Regulation & Welfare Loss

10.1 Profit Maximisation, Marginal Revenue, and the Lerner Index

A pure monopolist is the sole seller of a good without close substitutes, shielded by barriers to entry such as subadditive costs (natural monopoly), intellectual property patents, or statutory licences. Facing downward-sloping inverse market demand $P(Q)$ with $P'(Q) < 0$, total revenue is $R(Q) = P(Q) \cdot Q$, and marginal revenue is:

$$MR(Q) = \frac{dR}{dQ} = P(Q) + Q\frac{dP}{dQ} = P(Q)\left(1 + \frac{1}{\varepsilon}\right) = P(Q)\left(1 - \frac{1}{|\varepsilon|}\right)$$

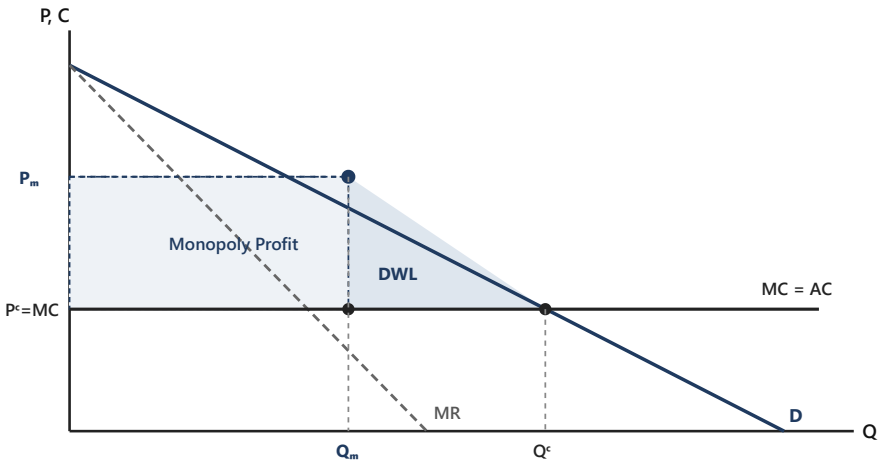
Where:

- $\varepsilon = \frac{dQ}{dP}\frac{P}{Q} < 0$ is the own-price elasticity of market demand, and $|\varepsilon| = -\varepsilon$.
- Because $QP'(Q) < 0$, $MR(Q) < P(Q)$ for all $Q > 0$: selling one more unit lowers the price on all inframarginal units.

Maximising $\pi(Q) = P(Q)Q - C(Q)$ requires $MR(Q_m) = MC(Q_m)$ (with $MR'(Q_m) < MC'(Q_m)$). Whenever $MC > 0$, optimality requires $MR > 0$, which immediately implies $|\varepsilon| > 1$: **a profit-maximising monopolist never operates on the inelastic portion of its demand curve** (because raising price when $|\varepsilon| < 1$ both increases revenue and reduces total cost). Rearranging $P_m(1 - 1/|\varepsilon|) = MC$ yields the **Lerner Index of Monopoly Power (L)**:

$$L = \frac{P_m - MC(Q_m)}{P_m} = \frac{1}{|\varepsilon|}$$

Where $L \in [0, 1)$ measures the proportional markup of price over marginal cost. In competitive markets, $|\varepsilon| \to \infty$ for an individual price-taking firm, giving $L = 0$. Equivalently, the markup pricing rule states $P_m = \frac{MC}{1 - 1/|\varepsilon|}$. Note that a monopolist has no supply curve independent of demand: a shift in demand elasticity can change P_m even without a change in MC.



10.2 Welfare Cost of Monopoly and Natural Monopoly Regulation

Because $P_m > MC(Q_m)$, a monopolist restricts output below the socially efficient level Q_c (where $P(Q_c) = MC(Q_c)$), creating a deadweight loss triangle. Under linear demand $P = a - bQ$ and constant marginal cost $MC = c$, $Q_m = (a-c)/(2b)$ is exactly half of competitive output $Q_c = (a-c)/b$, leaving a deadweight loss of $DWL = \frac{(a-c)^2}{8b}$—half the size of monopoly profit $\pi_m = \frac{(a-c)^2}{4b}$. Furthermore, Tullock (1967) observed that firms may expend real resources lobbying to acquire or preserve statutory monopoly rents (**rent-seeking**), potentially dissipating up to the entire profit rectangle in social waste.

A **natural monopoly** arises when a single firm's cost function is strictly subadditive over the relevant range of market demand ($C(Q) < \sum_{i=1}^{k} C(q_i)$ for $\sum q_i = Q$), typically because large fixed infrastructure costs (F) paired with low constant marginal cost (c) yield strictly declining average cost $AC(Q) = F/Q + c > MC(Q)$ everywhere. Examples in Australia include electricity transmission grids and regional water networks regulated by the Australian Energy Regulator (AER) and Essential Services Commission. Three regulatory remedies are standard:

1. **First-Best Marginal-Cost Pricing ($P = MC$):** Eliminates deadweight loss ($Q = Q_c$), yet forces the utility to incur a persistent financial loss equal to $-F$, requiring a taxpayer-funded lump-sum subsidy.
2. **Second-Best Average-Cost Pricing ($P = AC$):** Allows the utility to break even ($\pi = 0$) without public subsidy, though a smaller deadweight loss remains ($Q_{AC} < Q_c$).
3. **Regulated Two-Part Tariff ($T(q) = A + pq$):** Sets the usage charge equal to marginal cost ($p = MC$) to achieve allocative efficiency while recovering fixed infrastructure costs F through a flat connection access fee $A = F/N$ per household.

Worked Example 10.1: Monopoly Pricing, Lerner Index, DWL, and Specific vs Ad Valorem Taxes

A patented Australian medical diagnostics supplier faces inverse demand $P(Q) = 200 - 2Q$ and operates with constant marginal cost $MC = 40$ (with zero fixed cost).

1. **Step 1 — Profit-maximising price and quantity:** Marginal revenue is $MR(Q) = 200 - 4Q$. Setting $MR = MC$:

   $$200 - 4Q = 40 \implies 4Q = 160 \implies Q_m = 40$$

   Price is $P_m = 200 - 2(40) = 120$ dollars, yielding monopoly profit $\pi_m = (120 - 40)(40) = 3,200$ dollars.
2. **Step 2 — Lerner Index and Demand Elasticity:**

   $$L = \frac{120 - 40}{120} = \frac{2}{3} \approx 0.667$$

   Confirming $L = 1/|\varepsilon|$: at $P = 120, Q = 40$, $\varepsilon = \frac{dQ}{dP}\frac{P}{Q} = (-0.5)\frac{120}{40} = -1.5$, so $1/|\varepsilon| = 1/1.5 = 2/3$.
3. **Step 3 — Competitive benchmark and Deadweight Loss:** Setting $P = MC$ gives $200 - 2Q_c = 40 \implies Q_c = 80$ at $P_c = 40$ dollars. Thus:

$$DWL = \frac{1}{2}(P_m - MC)(Q_c - Q_m) = \frac{1}{2}(120 - 40)(80 - 40) = 1,600$$

The deadweight loss equals \$1,600.

4. **Step 4 — Pass-through of a specific tax t = \$20:** Effective marginal cost becomes $MC + t = 60$. Setting $200 - 4Q = 60$ gives $Q_t = 35$ and $P_t = 200 - 70 = 130$ dollars. Because $P_t - P_m = 10 = t/2$, linear monopoly passes through exactly 50% of a specific tax (unlike a constant-elasticity demand curve $Q = AP^{-|\varepsilon|}$, where $P_m = \frac{|\varepsilon|}{|\varepsilon|-1}MC$ implies *more than 100%* pass-through!).

Chapter 11: Price Discrimination and Non-Linear Pricing

11.1 Taxonomy and Feasibility Conditions

Price discrimination occurs when a firm with market power charges different prices for identical or nearly identical units of a good that cannot be explained by differences in marginal cost. Successful price discrimination requires two structural preconditions: (1) market power ($P > MC$), and (2) prevention of **consumer arbitrage** (resale between low-price and high-price buyers). Following Pigou (1920), microeconomics classifies price discrimination into three degrees:

Degree	Information Required	Pricing Mechanism	Efficiency & Surplus Outcome
First-Degree (Perfect)	Individual willingness to pay for every unit.	Personalised unit pricing or personalised two-part tariff.	$Q = Q_c, DWL = 0$; firm captures 100% of total surplus ($CS = 0$).
Second-Degree (Menu / Screening)	Distribution of buyer types (cannot observe individual type).	Self-selecting menus of quantity-price bundles, two-part tariffs, or quality tiers.	High-demand type gets efficient q_H^*; low-demand type is distorted downward ($q_L < q_L^{FB}$); high type keeps information rent.
Third-Degree (Group Pricing)	Observable segment indicator (age, student card, peak/off-peak time).	Uniform linear price p_k per segment k satisfying $MR_k = MC$.	Lower price to more elastic group; total welfare can rise or fall depending on total output change.

11.2 First-Degree and Second-Degree Screening (Incentive Compatibility)

Under **First-Degree Price Discrimination**, a monopolist charges each buyer their exact marginal willingness to pay along $P(Q)$—or equivalently charges each identical buyer a two-part tariff $T(q) = F + pq$ with unit price $p = MC$ and lump-sum entry fee $F = CS(MC)$. Because the marginal unit is priced at MC, output expands to the competitive level Q_c, eliminating deadweight loss entirely while transferring all surplus to the monopolist.

When consumers are heterogeneous (e.g., high-demand business travellers H with share λ and low-demand leisure travellers L with share $1 - \lambda$) and the firm cannot observe an individual's type directly, the firm must engage in **Second-Degree Price Discrimination (screening)**. Offering two bundles (q_L, T_L) and (q_H, T_H)—where T_i is total payment for quantity q_i and $V_i(q)$ is gross utility ($V_H(q) > V_L(q)$ and $V_H^{'}(q) > V_L^{'}(q)$ by the single-crossing property)—requires satisfying four constraints:

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- **Individual Rationality (IR_L, IR_H):** $V_L(q_L) - T_L \geq 0$ and $V_H(q_H) - T_H \geq 0$ (both types willingly participate).
- **Incentive Compatibility (IC_L, IC_H):** $V_L(q_L) - T_L \geq V_L(q_H) - T_H$ and $V_H(q_H) - T_H \geq V_H(q_L) - T_L$ (each type prefers its own designated package).

In the optimal screening contract, IR_L and IC_H bind ($T_L = V_L(q_L)$ and $T_H = V_H(q_H) - [V_H(q_L) - V_L(q_L)]$). Because raising q_L increases the mimicry surplus $V_H(q_L) - V_L(q_L)$ (the **information rent** that must be ceded to type H), the monopolist obeys two canonical principles:

1. **No distortion at the top:** Type H receives its first-best efficient quantity ($V_H^{'}(q_H^*) = MC$).
2. **Downward distortion at the bottom:** Type L receives strictly less than its first-best quantity ($V_L^{'}(q_L^*) > MC$) to limit the information rent extracted by type H.

11.3 Third-Degree Price Discrimination and the Inverse Elasticity Rule

When a firm sells to K identifiable, arbitrage-separated market segments with inverse demands $P_k(q_k)$ and common constant marginal cost c (or joint cost $C(\sum q_k)$), profit maximisation $\max_{q_1,\dots,q_K} \sum_{k=1}^{K} P_k(q_k)q_k - C(\sum q_k)$ sets marginal revenue in every active segment equal to common marginal cost:

$$MR_1(q_1) = MR_2(q_2) = \cdots = MR_K(q_K) = MC(Q)$$

Substituting $MR_k = p_k(1 - 1/|\varepsilon_k|)$ yields the **Inverse Elasticity Pricing Rule**:

$$p_1\left(1 - \frac{1}{|\varepsilon_1|}\right) = p_2\left(1 - \frac{1}{|\varepsilon_2|}\right) = MC \implies \frac{p_1}{p_2} = \frac{1 - 1/|\varepsilon_2|}{1 - 1/|\varepsilon_1|}$$

Where:

- $|\varepsilon_k| > 1$ is the absolute price elasticity of demand in segment k.
- The segment with the **higher elasticity** (more price-sensitive buyers, such as students or pensioners in Melbourne public transport) pays the **strictly lower price**.

Evaluating social welfare relative to uniform monopoly pricing, Schmalensee (1981) and Varian (1985) proved a sharp bound: $\Delta W \leq \sum_k (p_k - MC)\Delta q_k$ when demand shifts from uniform price p_0 to discriminatory prices p_k. Consequently, a **necessary condition** for third-degree price discrimination to increase total welfare is that **total output strictly increases** ($\sum \Delta q_k > 0$). If demand curves are linear and both markets are served under uniform pricing, total output is unchanged ($\Delta Q = 0$), so price discrimination strictly lowers welfare because misallocation across consumers ($p_1 \neq p_2$) is added without any output expansion.

Worked Example 11.1: Third-Degree Price Discrimination Across Two Segmented Markets

An Australian software firm sells statistical licences at constant marginal cost $MC = 20$ dollars to corporate analysts ($Q_1 = 100 - P_1$) and university researchers ($Q_2 = 60 - P_2$).

1. **Step 1 — Discriminatory prices and quantities:** Inverse demands are $P_1 = 100 - Q_1$ and $P_2 = 60 - Q_2$. Equating $MR_k = MC$:

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$$MR_1 = 100 - 2Q_1 = 20 \implies Q_1^* = 40, \quad P_1^* = 60$$

$$MR_2 = 60 - 2Q_2 = 20 \implies Q_2^* = 20, \quad P_2^* = 40$$

At these prices, $|\varepsilon_1| = 60/40 = 1.5$ (less elastic) and $|\varepsilon_2| = 40/20 = 2.0$ (more elastic), confirming $P_2^* < P_1^*$. Total profit is $\pi_{PD} = (60-20)(40) + (40-20)(20) = 1{,}600 + 400 = 2{,}000$ dollars, and total output is $Q_{PD} = 60$.

2. **Step 2 — Uniform pricing benchmark:** Summing demands for $P \le 60$ gives $Q = 160 - 2P$, or $P = 80 - 0.5Q$. Setting $MR = 80 - Q = 20$ yields $Q_U = 60$ and $P_U = 50$ dollars (serving $Q_1 = 50$ and $Q_2 = 10$). Uniform profit is $\pi_U = (50-20)(60) = 1{,}800$ dollars. Notice $Q_{PD} = Q_U = 60$, so welfare under discrimination falls because consumers no longer face equal marginal valuations ($P_1^* = 60 \neq P_2^* = 40$).

Worked Example 11.2: Second-Degree Screening with Two Consumer Types

A Sydney cloud-storage provider serves equal shares ($\lambda = 0.5$ each) of enterprise (H) and personal (L) users at zero marginal cost ($MC = 0$). Gross willingness to pay for q terabytes is $V_H(q) = 20q - q^2$ and $V_L(q) = 12q - q^2$.

1. **Step 1 — First-best quantities (if types were observable):** Setting $V_i'(q) = 0$ gives $q_H^{FB} = 10$ TB and $q_L^{FB} = 6$ TB.
2. **Step 2 — Screening objective with binding IR_L and IC_H:** Setting $T_L = V_L(q_L)$ and $T_H = V_H(q_H) - [V_H(q_L) - V_L(q_L)]$, where $V_H(q_L) - V_L(q_L) = 8q_L$ is the information rent paid to type H, expected profit is:

$$\mathrm{E}[\pi] = 0.5[20q_H - q_H^2 - 8q_L] + 0.5[12q_L - q_L^2]$$

3. **Step 3 — Optimal distorted menu:** Differentiating with respect to q_H and q_L:

$$\frac{\partial \mathrm{E}[\pi]}{\partial q_H} = 0.5(20 - 2q_H) = 0 \implies q_H^* = 10 \text{ TB (no distortion at the top)}$$

$$\frac{\partial \mathrm{E}[\pi]}{\partial q_L} = 0.5(-8 + 12 - 2q_L) = 0 \implies q_L^* = 2 \text{ TB (downward distortion from 6 to 2)}$$

The resulting fees are $T_L^* = 12(2) - 4 = 20$ dollars and $T_H^* = [20(10) - 100] - 8(2) = 100 - 16 = 84$ dollars, leaving enterprise users an information rent of \$16 per subscription.

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Chapter 12: Oligopoly Theory — Cournot, Bertrand, Stackelberg & Collusion

12.1 Cournot Quantity Competition and Industry Concentration

In an **oligopoly**, a small number of interdependent firms (such as Australia's major domestic airlines, supermarkets, or retail banks) make strategic decisions knowing each rival's action directly alters their own profit. Under **Cournot competition**, N firms simultaneously and independently select output quantities $q_1, \dots, q_N$ given inverse market demand $P(Q)$ where $Q = \sum_{i=1}^{N} q_i$. Firm i solves:

$$\max_{q_i \geq 0} \; \pi_i(q_i, q_{-i}) = P(q_i + Q_{-i})q_i - C_i(q_i)$$

Where $Q_{-i} = \sum_{j \neq i} q_j$ denotes rivals' combined output. Differentiating with respect to q_i holding Q_{-i} fixed yields the first-order condition and **Best-Response (Reaction) Function** $q_i = BR_i(Q_{-i})$:

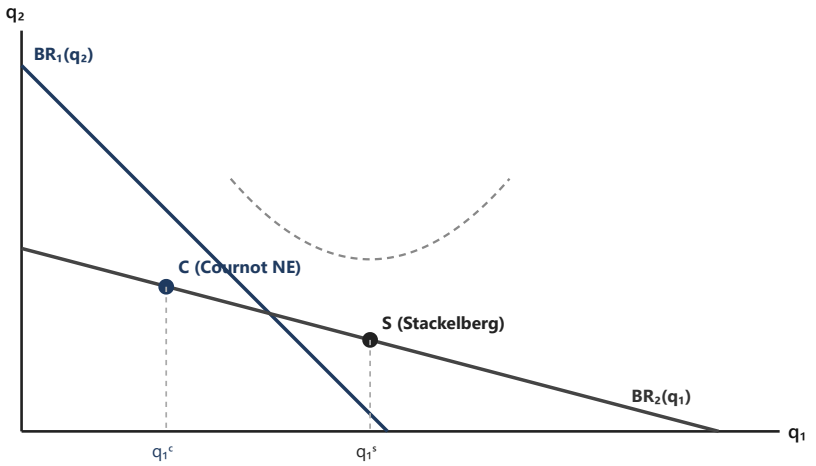
$$P(Q) + q_i P'(Q) = MC_i(q_i)$$

Where $P(Q) + q_i P'(Q)$ is firm i's perceived marginal revenue. A **Cournot-Nash equilibrium** is a vector $(q_1^*, \dots, q_N^*)$ such that $q_i^* = BR_i(Q_{-i}^*)$ simultaneously for all i. Rearranging the first-order condition in terms of market share $s_i = q_i/Q$ and market demand elasticity $|\varepsilon| = -\frac{P}{QP'(Q)}$ gives:

$$L_i = \frac{P - MC_i}{P} = \frac{s_i}{|\varepsilon|}, \qquad \sum_{i=1}^{N} s_i L_i = \frac{\sum_{i=1}^{N} s_i^2}{|\varepsilon|} = \frac{HHI}{|\varepsilon|}$$

Where $HHI = \sum_{i=1}^{N} s_i^2 \in (0, 1]$ (or 0 to $10,000$ in percentage-squared points used by the Australian Competition and Consumer Commission, ACCC) is the **Herfindahl-Hirschman Index**. For N symmetric firms with constant $MC = c$ and linear demand $P = a - bQ$, equilibrium firm output is $q^* = \frac{a-c}{b(N+1)}$, industry output is $Q^* = \frac{N}{N+1}\frac{a-c}{b}$, and price $P^* = \frac{a+Nc}{N+1}$ converges smoothly toward competitive marginal cost c as $N \to \infty$.

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12.2 Bertrand Price Competition and the Stackelberg Sequential Game

In **Bertrand competition**, firms compete simultaneously by setting prices (p_1, p_2) rather than quantities. When products are homogeneous, firms have identical constant marginal cost c, and capacity is unconstrained, buyers buy exclusively from the cheaper seller. If $p_1 > c$, firm 2 captures the entire market by undercutting to $p_2 = p_1 - \epsilon$. The unique Nash equilibrium is the **Bertrand paradox**: $p_1^* = p_2^* = c$, yielding zero economic profit with only two firms! Three realistic departures resolve the paradox: (1) asymmetric costs ($c_1 < c_2$, where firm 1 sets limit price $p_1 \approx c_2$), (2) capacity constraints (Kreps–Scheinkman shows capacity choice followed by price competition replicates the Cournot outcome), and (3) **product differentiation** (where upward-sloping price reaction curves $p_i(p_j)$ sustain positive equilibrium markups $p^* > c$).

In **Stackelberg quantity leadership**, Firm 1 (the Leader) commits to output q_1 first, and Firm 2 (the Follower) observes q_1 and chooses $q_2 = BR_2(q_1)$ in stage 2. Solving by **backward induction** to find the Subgame Perfect Nash Equilibrium (SPNE), the Leader substitutes $BR_2(q_1)$ directly into its profit function $\pi_1(q_1, BR_2(q_1))$ and maximises with respect to q_1. Because reaction curves slope downward (quantities are *strategic substitutes*), committing to a larger output ($q_1^S > q_1^C$) forces the Follower to contract ($q_2^S < q_2^C$), conferring a **first-mover advantage** ($\pi_1^S > \pi_1^C > \pi_2^S$).

12.3 Repeated Games, Tacit Collusion, and ACCC Cartel Policy

In a static one-shot Cournot or Bertrand game, a cartel agreement to split monopoly output Q_m is unstable because each firm has a unilateral incentive to cheat and expand output along its best-response curve (a Prisoner's Dilemma). However, when firms interact repeatedly over an infinite horizon with common discount factor $\delta \in (0, 1)$, **grim-trigger strategies**—cooperating at the collusive profit π^{Coll} so long as rivals never cheated, and reverting permanently to the competitive Nash profit π^{NE} upon any deviation—can sustain collusion if the present value of cooperation exceeds the one-period deviation gain π^D:

$$\frac{\pi^{Coll}}{1-\delta} \geq \pi^D + \frac{\delta \pi^{NE}}{1-\delta} \iff \delta \geq \delta^* = \frac{\pi^D - \pi^{Coll}}{\pi^D - \pi^{NE}}$$

Where:

- $\delta \in (0, 1)$ is the common per-period discount factor ($\delta = \frac{1}{1+r}$).

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- π^{Coll}, π^{D}, and π^{NE} denote per-period collusive, optimal unilateral deviation, and static Nash equilibrium profits, respectively.

Under Part IV of Australia's *Competition and Consumer Act 2010 (Cth)*, explicit price-fixing, output restriction, and market-sharing cartels are strictly illegal civil and criminal offences enforced by the ACCC (supported by the ACCC's immunity policy for first-reporting cartel whistleblowers, which destabilises trust and raises the effective threshold δ^{*}).

Market Regime (Symmetric Linear: $P = a - bQ, MC = c$)	Industry Output Q	Market Price P	Total Industry Profit Π
Competitive / Homogeneous Bertrand ($N \geq 2$)	$\frac{a-c}{b}$	c	0
Stackelberg Duopoly ($N = 2$)	$\frac{3}{4}\frac{a-c}{b}$	$\frac{a+3c}{4}$	$\frac{3}{16}\frac{(a-c)^2}{b}$
Cournot Duopoly ($N = 2$)	$\frac{2}{3}\frac{a-c}{b}$	$\frac{a+2c}{3}$	$\frac{2}{9}\frac{(a-c)^2}{b}$
Monopoly / Joint Cartel	$\frac{1}{2}\frac{a-c}{b}$	$\frac{a+c}{2}$	$\frac{1}{4}\frac{(a-c)^2}{b}$

Worked Example 12.1: Cournot, Stackelberg, Differentiated Bertrand & Collusion Stability

Two Australian regional aviation carriers compete on a route with inverse demand $P = 180 - Q$ (where $Q = q_1 + q_2$ in thousands of seats) and constant marginal cost $c = 60$ dollars per seat.

1. **Step 1 — Cournot duopoly equilibrium:** Firm 1 maximises $(180 - q_1 - q_2 - 60)q_1 = (120 - q_1 - q_2)q_1$. Setting $\frac{\partial \pi_1}{\partial q_1} = 120 - 2q_1 - q_2 = 0$ yields $BR_1(q_2) = 60 - 0.5q_2$. By symmetry, $BR_2(q_1) = 60 - 0.5q_1$. Solving $q_1 = 60 - 0.5(60 - 0.5q_1)$ gives:

   > $$q_1^C = q_2^C = 40, \quad Q^C = 80, \quad P^C = 100, \quad \pi_1^C = \pi_2^C = (100 - 60)(40) = 1,600$$

   Each carrier earns $1,600 thousand ($1.6 million).
2. **Step 2 — Stackelberg leadership (Carrier 1 moves first):** Carrier 1 anticipates $q_2 = 60 - 0.5q_1$ and maximises:

   > $$\pi_1(q_1) = (180 - q_1 - (60 - 0.5q_1) - 60)q_1 = (60 - 0.5q_1)q_1$$

   Setting $\frac{d\pi_1}{dq_1} = 60 - q_1 = 0$ gives $q_1^S = 60$, so $q_2^S = 60 - 0.5(60) = 30$, $Q^S = 90$, and $P^S = 90$ dollars. Profits are $\pi_1^S = (90 - 60)(60) = 1,800$ (up from 1,600) and $\pi_2^S = (90 - 60)(30) = 900$ (down from 1,600).
3. **Step 3 — Collusion and critical discount factor (under repeated Cournot):** Joint monopoly output solves $180 - 2Q = 60 \implies Q_m = 60$, so symmetric cartel shares are $q_1^{Coll} = q_2^{Coll} = 30$, $P^{Coll} = 120$, and $\pi^{Coll} = (120 - 60)(30) = 1,800$. If Carrier 1 deviates while Carrier 2 produces 30, Carrier 1 plays $BR_1(30) = 60 - 0.5(30) = 45$, yielding $P = 180 - 75 = 105$ and $\pi^{D} = (105 - 60)(45) = 2,025$. With $\pi^{NE} = 1,600$, grim-trigger collusion is sustainable if and only if:

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$$\delta \geq \frac{2,025-1,800}{2,025-1,600} = \frac{225}{425} = \frac{9}{17} \approx 0.5294$$

By contrast, under repeated homogeneous Bertrand competition ($\pi^{NE} = 0$, $\pi^{Coll} = 1,800$, and $\pi^{D} \approx 3,600$ by slightly undercutting $P_m = 120$), the critical discount factor simplifies to $\delta \geq \frac{3,600-1,800}{3,600-0} = 0.50$.

4. **Step 4 — Differentiated Bertrand check:** Suppose instead the two carriers offer differentiated departure schedules with demand $q_i = 60 - p_i + 0.5p_j$ and $c = 20$. Maximising $(p_i - 20)(60 - p_i + 0.5p_j)$ gives $60 - 2p_i + 0.5p_j + 20 = 0 \implies p_i = 40 + 0.25p_j$. Notice price reaction curves slope *upward* (prices are *strategic complements*). Symmetric equilibrium yields $p_1^* = p_2^* = 40/0.75 = 53.33$ dollars, $q_i^* = 33.33$, and $\pi_i^* = 1,111.11$—resolving the Bertrand paradox cleanly.

12.4 Mergers, Market Power & Upward Pricing Pressure (GUPPI) under ACCC Guidelines

Under Section 50 of the *Competition and Consumer Act 2010* (Cth), the Australian Competition and Consumer Commission (ACCC) assesses whether proposed corporate acquisitions or mergers substantially lessen competition across relevant Australian markets. Traditional merger enforcement relied primarily on market share concentration thresholds, specifically the **Herfindahl-Hirschman Index (HHI)**:

$$\text{HHI} = sum_{i=1}^{N} s_i^2, \qquad Delta\text{HHI} = 2s_1 s_2$$

Where s_i represents the percentage market share of firm i (such that $sums_i = 100$), and $Delta$HHI is the post-merger change in concentration resulting from the union of firms with market shares s_1 and s_2. Under ACCC merger guidelines, an acquisition raising HHI by more than 100 points in an industry with post-merger HHI exceeding 2,000 triggers rigorous forensic investigation.

In modern differentiated-products markets (such as retail banking, telecommunications, and supermarket retailing), market shares fail to capture direct product substitution. Regulators evaluate unilateral post-merger price incentives using the **Gross Upward Pricing Pressure Index (GUPPI)**:

$$\text{GUPPI}_1 = D_{12}\left(\frac{p_2}{p_1}\right) m_2$$

Where:

GUPPI_1 measures the proportional upward pricing incentive on Product 1 post-merger;

$D_{12} = -\frac{\partial q_2/\partial p_1}{\partial q_1/\partial p_1}$ is the diversion ratio from Product 1 to Product 2 (the fraction of lost sales from raising Product 1's price captured by Product 2);

p_1 and p_2 are the respective pre-merger retail prices;

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$m_2 = \frac{p_2 - c_2}{p_2}$ is the pre-merger percentage gross margin of Product 2.

Worked Numerical Example: ACCC Telecommunications Merger Screening

Two Australian retail mobile network operators, Telco A and Telco B, propose a merger. Prior to the merger, Telco A sells an unlimited 5G mobile plan for $p_1 = 60$ AUD per month with marginal cost $c_1 = 15$ AUD (margin $m_1 = (60 - 15)/60 = 75.0\%$). Telco B sells a competing plan for $p_2 = 50$ AUD per month with marginal cost $c_2 = 20$ AUD (margin $m_2 = (50 - 20)/50 = 60.0\%$). Econometric demand estimation reveals a diversion ratio of $D_{12} = 0.25$ (25% of customers abandoning Telco A switch directly to Telco B).

Compute GUPPI_1 and determine the minimum percentage marginal cost reduction (merger efficiency) required to completely offset unilateral price increases.

$$\text{GUPPI}_1 = 0.25 \times (\frac{50}{60}) \times 0.60 = 0.25 \times 0.8333 \times 0.60 = 0.1250 \text{ (or } 12.50\%)$$

Telco A faces an immediate 12.50% upward pricing incentive, substantially exceeding the ACCC's standard 5.0% enforcement threshold. For linear demand, the required compensating marginal cost reduction E_1 is:

$$E_1 = \frac{\text{GUPPI}_1}{1 - m_1} = \frac{0.1250}{1 - 0.75} = \frac{0.1250}{0.25} = 50.0\%$$

Telco A would need to reduce network operating costs by 50.0% to neutralise price hikes, leading the ACCC to reject the merger absent substantial divestiture remedies.

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