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Bachelor of Commerce (B2001) | Level 3 • Part B Major | Monash University

Investments

Course Code: BFC3241 • Module 1 Notes

Module 1: Asset Classes, Financial Markets, Return Measurement & Risk-Return Tradeoff

1.1 The Investment Environment: Real Assets, Financial Assets, and Australian Market Architecture

Investment analysis examines how households, superannuation funds, endowments, and corporate treasuries defer current consumption to acquire claims on future purchasing power. A foundational distinction separates **real assets** from **financial assets**. Real assets consist of the land, buildings, plant, machinery, intellectual property, and human capital that directly generate goods and services within the economy. By contrast, financial assets—such as ordinary shares, Australian Commonwealth Government Bonds (ACGBs), bank bills, and derivative contracts—are paper or electronic claims against the income and cash flows produced by real assets. Whereas real assets determine an economy's aggregate productive capacity, financial assets determine how ownership of that capacity and its associated risks are allocated across investors.

Financial markets perform three indispensable economic functions: consumption timing across the life cycle, allocation of risk to investors most willing to bear it, and separation of corporate ownership from day-to-day management. In Australia, capital formation and secondary trading occur across an integrated institutional ecosystem supervised under a "twin peaks" regulatory model:

- **Reserve Bank of Australia (RBA):** Conducts monetary policy by setting the target overnight cash rate, manages systemic liquidity through open market operations and repurchase agreements, and oversees clearing and settlement stability under the *Payments Systems (Regulation) Act 1998 (Cth)*.
- **Australian Prudential Regulation Authority (APRA):** Supervises Authorised Deposit-taking Institutions (ADIs), general and life insurers, and registrable superannuation entities (RSEs) to ensure capital adequacy, liquidity resilience, and fiduciary governance.
- **Australian Securities and Investments Commission (ASIC):** Enforces market integrity, corporate disclosure, financial services licensing (AFSL), and consumer protection under the *Corporations Act 2001 (Cth)* and the *ASIC Act 2001 (Cth)*.
- **Market Operators and Infrastructure:** The Australian Securities Exchange (ASX) operates the primary equity and ASX 24 derivatives platforms, Cboe Australia provides competing multi-asset equity trading, and settlement occurs via CHESS (equities, $T + 2$ settlement) and Austraclear (wholesale debt and money market securities).

Markets are further categorised into **primary markets**, where issuers raise new capital via Initial Public Offerings (IPOs), rights issues, placements, or bond tenders conducted by the Australian Office of Financial Management (AOFM), and **secondary markets**, where existing securities trade among investors without transferring new cash to the issuer. Secondary market liquidity lowers the cost of capital in the primary market because investors demand a smaller illiquidity premium when they can exit positions rapidly at low transaction cost.

1.2 Money Market, Fixed-Income, Equity, and Derivative Asset Classes

Institutional asset managers group investable securities into distinct asset classes based on cash-flow structure, maturity, legal seniority, and risk-return profile. Within Australian portfolios, allocations span cash and money market instruments, domestic and global fixed income, listed equities, property and infrastructure, and derivatives or alternative strategies.

Asset Class	Australian Benchmarks & Instruments	Cash-Flow Profile	Primary Risk Exposures
Cash & Money Market	RBA Cash Rate, 90-day Bank-Accepted Bills (BBSW), NCDs, Treasury Notes	Short-term (≤ 1 year), zero-coupon discount securities	Reinvestment risk, minimal credit and duration risk

Asset Class	Australian Benchmarks & Instruments	Cash-Flow Profile	Primary Risk Exposures
Fixed Income (Bonds)	ACGBs, Semi-Government Bonds (e.g., TCorp, TCV), Corporate MTNs, TIBs	Semi-annual fixed or inflation-indexed coupons plus face value	Interest rate (duration), inflation, credit spread, and liquidity risk
Equities	S&P/ASX 200, S&P/ASX 300, Small Ordinaries, franked ordinary shares	Residual earnings, discretionary dividends + franking credits	Systematic market risk, earnings volatility, idiosyncratic risk
Listed Property & Infrastructure	S&P/ASX 200 A-REITs, toll-road and utility trusts	Rental/concession distributions, inflation-linked revenues	Cap-rate expansion, leverage, regulatory tariff resets
Derivatives & Alternatives	ASX 24 SPI 200 Futures, ETOs, private equity, hedge funds	Contingent payoffs, asymmetric or leveraged exposures	Basis risk, counterparty risk, illiquidity, model/leverage risk

Money Market Conventions in Australia

Money market securities are high-liquidity, low-default-risk debt instruments with original maturities of one year or less. In Australia, wholesale short-term borrowing rates are anchored by the **Bank Bill Swap Rate (BBSW)**, administered by ASX Benchmarks and derived from observable transactions in prime bank bills and Negotiable Certificates of Deposit (NCDs). Unlike coupon bonds, Australian Treasury Notes, Bank-Accepted Bills (BABs), and commercial paper are issued as **pure discount securities**: they pay no periodic interest and instead trade below face value F, maturing at F. Under Australian money market convention, prices are quoted using simple (add-on) yield on an Actual/365 day-count basis:

$$P = \frac{F}{1 + r_{\text{simple}} \times \frac{d}{365}}$$

Where: P is the purchase price of the bill in dollars; F is the face value repaid at maturity; r_{simple} is the quoted annual simple money market yield; and d is the exact number of days remaining until maturity.

By contrast, United States Treasury bills are quoted on a bank discount yield basis, which divides the dollar discount by face value F and uses a 360-day year: $r_{\text{BD}} = \frac{F-P}{F} \times \frac{360}{d}$. Because Australian institutional investors compare money market instruments against annualised bond yields and cash accounts, a simple money market yield must be converted into an **Effective Annual Rate (EAR)** that accounts for intra-year compounding:

$$\text{EAR} = \left(1 + r_{\text{simple}} \times \frac{d}{365}\right)^{\frac{365}{d}} - 1 = \left(\frac{F}{P}\right)^{\frac{365}{d}} - 1$$

Where: EAR is the compounded effective annual rate; F/P is the holding-period gross return factor over d days; and $365/d$ is the number of compounding periods per year.

Worked Example 1: Pricing an Australian Bank-Accepted Bill and Computing EAR

A corporate treasury purchases a 90-day Bank-Accepted Bill with a face value of $500,000 quoted at a simple yield of 4.65% p.a. (Actual/365). Calculate the purchase price, the dollar interest earned, and the Effective Annual Rate (EAR).

1. **Compute the purchase price using the Australian Actual/365 discount formula:**

$$P = \frac{\$500,000}{1 + 0.0465 \times \frac{90}{365}} = \frac{\$500,000}{1 + 0.01146575} = \frac{\$500,000}{1.01146575} = \$494,332.11$$

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2. **Determine the dollar interest earned over the 90-day holding period:**

$$\text{Interest} = F - P = \$500,000 - \$494,332.11 = \$5,667.89$$

3. **Convert the 90-day return to an Effective Annual Rate (EAR):**

$$\text{EAR} = (1 + 0.01146575)^{\frac{365}{90}} - 1 = (1.01146575)^{4.055556} - 1 = 1.047323 - 1 = 4.7323\%$$

Note that the EAR (4.7323%) exceeds the quoted simple yield (4.65%) because interest earned every 90 days can be reinvested roughly 4.056 times per year.

Equities, Franking Credits, and Fixed-Income Instruments

Ordinary shares represent residual ownership claims on corporate cash flows. Investors earn returns through capital appreciation and dividends. A distinctive institutional feature of Australian equity investing is the **dividend imputation system** introduced in 1987. Under Australian tax law, when an ASX-listed company pays corporate tax at the statutory rate T_c (typically 30% for large listed firms), it attaches **franking credits** to dividends distributed from taxed profits, preventing double taxation of corporate income for resident shareholders. If a company pays a cash dividend D_{cash} franked to proportion $f \in [0, 1]$, the attached franking credit is:

$$FC = D_{\text{cash}} \times \frac{T_c}{1 - T_c} \times f$$

Where: FC is the franking credit attached to the dividend; D_{cash} is the cash dividend received; T_c is the corporate tax rate (0.30 for large companies); and f is the franking percentage (1.00 for a fully franked dividend).

A domestic tax-exempt superannuation fund receives a full cash refund of FC, whereas an individual investor on marginal tax rate T_p is taxed on the grossed-up dividend $D_{\text{cash}} + FC$ and receives a tax offset equal to FC. Consequently, portfolio managers evaluating Australian shares frequently report both cash dividend yield and **grossed-up (franking-adjusted) yield**. Consider a fully franked cash dividend of \$70.00 per share when $T_c = 30\%$: the attached franking credit is $\$70.00 \times \frac{0.30}{0.70} = \$30.00$, giving a grossed-up dividend of \$100.00. For a complying superannuation fund in pension phase ($T_p = 0\%$), the after-tax cash inflow is \$100.00; for an accumulation-phase superannuation fund ($T_p = 15\%$), tax on the grossed-up dividend is \$15.00 offset by the \$30.00 credit, yielding a \$15.00 refund and \$85.00 after-tax cash; for an individual at the 47% top marginal rate (including Medicare levy), tax is \$47.00 less the \$30.00 offset, leaving \$53.00 after tax.

Fixed-income markets provide contractual debt claims ranging from default-free Commonwealth Government Bonds (ACGBs) and state-issued semi-government bonds (such as NSW TCorp and Treasury Corporation of Victoria) to investment-grade corporate Medium-Term Notes (MTNs) and ASX-listed bank hybrid securities (Additional Tier 1 capital notes). Unlike senior bonds, Australian bank AT1 hybrids contain non-viability and Common Equity Tier 1 (CET1) capital triggers mandated by APRA under Basel III, causing them to convert into ordinary equity or suffer write-down during banking stress—producing equity-like downside tail risk despite floating-rate distributions.

Pooled Investment Vehicles, ETFs, and Derivative Markets

Investors access asset classes either directly or through pooled vehicles regulated under Chapter 5C of the *Corporations Act 2001 (Cth)* as Managed Investment Schemes (MIS):

- **Unlisted Managed Funds (Unit Trusts):** Open-ended vehicles priced once daily at Net Asset Value (NAV). Investors subscribe and redeem units directly with the Responsible Entity at forward-priced NAV adjusted for a buy-sell spread.

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- **Exchange-Traded Funds (ETFs):** Open-ended index or active funds traded continuously on the ASX or Cboe Australia. Authorised Participants (APs) arbitrage deviations between the ETF's market price and its intraday indicative NAV (iNAV) via in-specie creation and redemption baskets, keeping market prices tightly tethered to underlying asset values.
- **Listed Investment Companies (LICs) and Listed Investment Trusts (LITs):** Closed-end companies or trusts with a fixed share count. Because shares cannot be created or redeemed on demand, LICs frequently trade at persistent premiums or discounts to pre-tax Net Tangible Assets (NTA), reflecting manager reputation, fee drag, and retained franking credit balances.
- **Derivative Instruments (Futures and Options):** On ASX 24, the SPI 200 equity index futures contract (valued at $25 per index point) and 3-year/10-year Australian Treasury bond futures allow portfolio managers to adjust systematic risk exposure rapidly at minimal transaction cost through daily mark-to-market variation margining. Exchange-Traded Options (ETOs) provide asymmetric payoff profiles—call options confer the right to buy at a strike price K, whereas put options confer the right to sell at K, enabling protective portfolio insurance and covered-call income overlays.

Empirical long-run evidence across Australian capital markets (1900–present) confirms a persistent hierarchy of average annual returns and volatilities: Australian equities have delivered higher long-run compound real returns than Australian government bonds, which in turn have outperformed Treasury bills and cash—reflecting the positive **equity risk premium** and **term premium** demanded by risk-averse investors.

1.3 Trading Mechanics, Market Microstructure, and Margin Trading

Security prices observed on screens reflect not only fundamental news but also market microstructure—the rules governing order submission, execution priority, liquidity provision, and settlement. On the ASX, continuous trading operates via an electronic Central Limit Order Book (CLOB) using price-time priority, flanked by opening and closing single-price call auctions that concentrate liquidity and establish official closing prices for index funds.

- **Market Orders vs Limit Orders:** A *market order* executes immediately against the best available resting quotes on the opposite side of the book, guaranteeing execution speed while exposing the trader to price uncertainty and the bid-ask spread. A *limit order* specifies a maximum purchase price or minimum sale price, providing price protection and supplying liquidity, though execution is uncertain if the market moves away.
- **Stop-Loss and Stop-Limit Orders:** A *stop-loss order* converts into a market order once a trigger price is breached, whereas a *stop-limit order* converts into a limit order—protecting against adverse execution slippage in thin markets at the risk of non-execution during a gap down.
- **Bid-Ask Spread and Market Depth:** Dealers and limit-order traders quote a bid price (the highest price buyers will pay) and an ask price (the lowest price sellers will accept). The bid-ask spread measures the immediate round-trip liquidity cost of trading. Institutional block trades frequently execute in dark pools (e.g., ASX Centre Point) at the midpoint of the National Best Bid and Offer (NBBO) to minimise market impact.

Buying on Margin and Short Selling

Investors can amplify expected returns—and potential losses—by borrowing funds from a broker to purchase shares (**buying on margin**) or by borrowing shares to sell them with the expectation of repurchasing at a lower price (**short selling**). Margin accounts are governed by two equity thresholds: the **initial margin** (m_0), representing the minimum percentage of the position's value funded with the investor's own equity at inception, and the **maintenance margin** (m_m), representing the minimum equity ratio required on an ongoing basis to prevent a margin call.

For a long margin purchase of N shares at initial price P_0 with initial margin m_0, the investor contributes equity $E_0 = m_0 N P_0$ and borrows $L = (1 - m_0) N P_0$. If the share price moves to P_1, the account equity becomes $N P_1 - L$ (ignoring margin interest for simplicity, or $N P_1 - L(1 + r_{\text{loan}})$ when interest accrues). The percentage equity margin is $(N P_1 - L)/(N P_1)$. Setting this ratio equal to the maintenance margin m_m yields the **long margin call trigger price** $P_{\text{call}}^{\text{long}}$:

$$P_{\text{call}}^{\text{long}} = \frac{L}{N(1 - m_m)} = P_0 \left(\frac{1 - m_0}{1 - m_m}\right)$$

Where: $P_{\text{call}}^{\text{long}}$ is the share price below which a margin call occurs; P_0 is the initial purchase price; L is the margin loan balance; N is the number of shares; m_0 is the initial margin ratio; and m_m is the maintenance margin ratio.

In a **short sale**, an investor borrows N shares from a securities lender (often a large superannuation custodian), sells them immediately at P_0, and deposits initial margin $m_0 N P_0$ alongside the sale proceeds $N P_0$ as collateral, yielding total account assets of $N P_0(1 + m_0)$. At any future price P_1, the short seller's equity is $N P_0(1 + m_0) - N P_1$ and the liability is $N P_1$. Setting equity divided by liability equal to m_m gives the **short-sale margin call trigger price** $P_{\text{call}}^{\text{short}}$:

$$P_{\text{call}}^{\text{short}} = P_0 \left(\frac{1 + m_0}{1 + m_m}\right)$$

Where: $P_{\text{call}}^{\text{short}}$ is the share price above which the short seller receives a margin call; P_0 is the initial short-sale price; m_0 is the initial margin; and m_m is the maintenance margin.

Note a fundamental asymmetry between long and short positions: a long investor's maximum loss is bounded at 100% of invested capital (if the share price falls to zero), whereas a short seller faces theoretically unlimited loss if the share price surges sharply, creating vulnerability to a **short squeeze** when forced buy-ins drive prices even higher. In Australia, ASIC enforces strict short-selling reporting rules under the *Corporations Act 2001 (Cth)* s 1020B, prohibiting naked short selling and requiring daily aggregate short-position disclosures.

Worked Example 2: Margin Purchase and Short-Sale Margin Call

An investor expects BHP Group shares, currently trading at $45.00 on the ASX, to appreciate. The broker requires an initial margin of 50% and a maintenance margin of 30%. The investor buys 1,000 BHP shares on margin at $45.00. Assume margin loan interest is 7.00% p.a. over a 1-year horizon and BHP pays an annual cash dividend of $2.20 per share.

1. **Determine initial position values:**

$$\text{Position Value} = 1{,}000 \times \$45.00 = \$45{,}000, \quad \text{Initial Equity} = 0.50 \times \$45{,}000 = \$22{,}500$$

$$\text{Margin Loan} = \$45{,}000 - \$22{,}500 = \$22{,}500$$

2. **Calculate the immediate margin call trigger price (before interest accrual):**

$$P_{\text{call}}^{\text{long}} = \$45.00 \times \left(\frac{1 - 0.50}{1 - 0.30}\right) = \$45.00 \times \frac{0.50}{0.70} = \$32.14$$

If BHP falls below $32.14, account equity drops below 30% of market value and the broker issues a margin call.

3. **Evaluate the 1-year levered Rate of Return if BHP rises to $52.00:**

$$\text{Ending Share Value} = 1{,}000 \times \$52.00 = \$52{,}000, \quad \text{Dividends} = 1{,}000 \times \$2.20 = \$2{,}200$$

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$$\text{Loan Repayment with Interest} = \$22,500 \times 1.07 = \$24,075$$

$$\text{Ending Equity} = \$52,000 + \$2,200 - \$24,075 = \$30,125$$

$$r_{\text{margin}} = \frac{\$30,125 - \$22,500}{\$22,500} = \frac{\$7,625}{\$22,500} = 33.89\%$$

Compare this to the unlevered return of $(\$52.00 - \$45.00 + \$2.20)/\$45.00 = 20.44\%$. Financial leverage magnifies returns whenever the asset return exceeds the borrowing cost, and magnifies losses symmetrically when returns fall short.

4. **Compare with a short sale of 1,000 shares at $45.00 (50% initial, 30% maintenance margin):**

$$P_{\text{call}}^{\text{short}} = \$45.00 \times \left(\frac{1 + 0.50}{1 + 0.30}\right) = \$45.00 \times \frac{1.50}{1.30} = \$51.92$$

If BHP rises to $52.00 instead of falling, the short seller breaches the maintenance threshold and faces a margin call.

1.4 Return Measurement, Multi-Period Aggregation, and Risk Metrics

Accurate performance measurement begins with the single-period **Holding Period Return (HPR)**, which captures both income distributions (dividends or coupons) and capital gains or losses over the measurement interval:

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0} = \underbrace{\frac{D_1}{P_0}}_{\text{Income Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gains Yield}}$$

Where: P_0 is the beginning security price; P_1 is the ending security price; and D_1 is the cash distribution received during the period (plus franking credits if measuring pre-tax grossed-up returns for Australian superannuation or resident portfolios).

Arithmetic Mean, Geometric Mean, Time-Weighted, and Dollar-Weighted Returns

When evaluating historical performance over T sub-periods with returns $r_1, r_2, \ldots, r_T$, portfolio analysts distinguish between the **arithmetic mean** ($\bar{r}_A$) and the **geometric mean** ($\bar{r}_G$, also called the Compound Annual Growth Rate or Time-Weighted Rate of Return when there are no interim cash flows):

$$\bar{r}_A = \frac{1}{T}\sum_{t=1}^{T} r_t, \qquad \bar{r}_G = \left[\prod_{t=1}^{T}(1 + r_t)\right]^{\frac{1}{T}} - 1$$

Where: r_t is the holding period return in period t; and T is the total number of periods.

Because compounding is multiplicative, volatility creates a wedge between arithmetic and geometric averages, approximated by $\bar{r}_G \approx \bar{r}_A - \frac{1}{2}\sigma^2$. The arithmetic mean provides an unbiased estimate of the expected single-period return (assuming returns are independent and identically distributed), making it appropriate for discounting one-period expectations and computing Sharpe ratios. By contrast, the geometric mean measures the actual compound rate at which wealth grew over the sample period.

When investors deposit or withdraw capital during the evaluation window, two distinct performance metrics are required:

- **Time-Weighted Rate of Return (TWRR):** Chains sub-period holding period returns calculated immediately before and after every external cash flow, eliminating the distortion caused by investor deposits or redemptions. Under CFA Institute Global Investment Performance Standards (GIPS), TWRR is the mandatory benchmark for evaluating active fund managers because managers do not control when clients contribute or withdraw money.
- **Dollar-Weighted Rate of Return (DWRR / Money-Weighted Rate of Return):** Equals the Internal Rate of Return (IRR) that sets the present value of all cash inflows and outflows (including initial investment and ending portfolio value) to zero: $\sum_{t=0}^{T} \frac{CF_t}{(1+\text{IRR})^t} = 0$. DWRR measures the actual return earned by the investor on dollars at work; if an investor injects heavy capital right before a market downturn, DWRR falls significantly below TWRR.

Worked Example 3: Arithmetic, Geometric, TWRR, and Dollar-Weighted Returns

An Australian equity investor deposits \$100,000 at $t = 0$ into an ASX 200 mandate. In Year 1, the portfolio earns +20.0%. At $t = 1$ (after Year 1 gains), the investor contributes an additional \$100,000. In Year 2, the market corrects and the portfolio earns -15.0%. At $t = 2$, the investor withdraws \$50,000. In Year 3, the portfolio earns +12.0%. Compare the arithmetic mean, geometric mean (TWRR), and Dollar-Weighted Return (IRR).

Period / Event	Beginning Value (\$)	Sub-Period Return	Value Before Cash Flow (\$)	External Cash Flow (\$)	Ending Value After CF (\$)
$t = 0$ Initial Deposit	0	—	0	+100,000	100,000
Year 1 ($t = 1$)	100,000	+20.0%	120,000	+100,000	220,000
Year 2 ($t = 2$)	220,000	-15.0%	187,000	-50,000	137,000
Year 3 ($t = 3$)	137,000	+12.0%	153,440	—	153,440

1. **Compute the Arithmetic Mean of annual returns:**

$$\bar{r}_A = \frac{0.20 + (-0.15) + 0.12}{3} = \frac{0.17}{3} = 5.67\% \text{ p.a.}$$

2. **Compute the Geometric Mean (which equals the Time-Weighted Rate of Return, TWRR):**

$$\text{TWRR} = [(1.20)(0.85)(1.12)]^{\frac{1}{3}} - 1 = (1.1424)^{\frac{1}{3}} - 1 = 1.04538 - 1 = 4.54\% \text{ p.a.}$$

3. **Compute the Dollar-Weighted Return (IRR) from the investor's cash-flow stream:** From the investor's perspective, outflows into the fund are negative and inflows back to the investor (including terminal liquidation value at $t = 3$) are positive: $CF_0 = -\$100,000$, $CF_1 = -\$100,000$, $CF_2 = +\$50,000$, $CF_3 = +\$153,440$. Setting NPV to zero:

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$$-100,000 - \frac{100,000}{1 + r_{\text{DW}}} + \frac{50,000}{(1 + r_{\text{DW}})^2} + \frac{153,440}{(1 + r_{\text{DW}})^3} = 0$$

Testing $r_{\text{DW}} = 0.0070$ (0.70%):

$$-100,000 - 99,304.87 + 49,307.28 + 150,262.18 = +\$264.59$$

Testing $r_{\text{DW}} = 0.0076$ (0.76%):

$$-100,000 - 99,245.73 + 49,248.55 + 149,993.96 = -\$3.22 \approx 0$$

Hence, $r_{\text{DW}} \approx 0.76\%$ p.a. Notice why DWRR (0.76%) is dramatically below TWRR (4.54%): the investor doubled the capital base to $220,000 immediately prior to the -15.0% drawdown in Year 2 and withdrew $50,000 before the +12.0% rebound in Year 3.

Real vs Nominal Returns and Downside Risk Measures

Investors care about real purchasing power rather than nominal dollar balances. If nominal return is R and Consumer Price Index (CPI) inflation is i, the exact **Fisher Equation** links nominal and real returns r:

$$1 + R = (1 + r)(1 + i) \implies r = \frac{1 + R}{1 + i} - 1 \approx R - i$$

Where: R is the nominal rate of return; r is the real (inflation-adjusted) rate of return; and i is the realised or expected inflation rate.

Risk is most commonly quantified via **variance** (σ^2) and **standard deviation** (σ). For forward-looking scenario analysis across states $s = 1, \ldots, S$ with probabilities $p(s)$, and for historical time-series samples of length T (using Bessel's correction $T - 1$ for sample variance):

$$\sigma^2_{\text{ex-ante}} = \sum_{s=1}^{S} p(s)\,[r(s) - E(r)]^2\,, \qquad \hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2$$

Where: $p(s)$ is the probability of economic state s; $r(s)$ is the return in state s; $E(r)$ is expected return; r_t is realised return in period t; and $\bar{r}$ is the sample mean return.

Standard deviation treats upside surprises and downside losses symmetrically. Because institutional investors are disproportionately concerned with left-tail losses—especially when equity returns exhibit negative skewness and excess kurtosis ("fat tails")—practitioners supplement volatility with downside metrics:

- **Lower Partial Standard Deviation (Semi-Deviation) and Sortino Ratio:** Computes dispersion solely over observations falling below a Minimum Acceptable Return (MAR), typically r_f or 0%. The Sortino ratio replaces total σ in the Sharpe denominator with downside deviation σ_D.
- **Value at Risk (VaR):** Estimates the loss threshold exceeded only with small probability α (e.g., 5% or 1%) over a specified horizon. Under normality, $\text{VaR}_\alpha = -(\mu - z_\alpha \sigma)$, where $z_{0.05} = 1.645$ and $z_{0.01} = 2.326$.

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- **Expected Shortfall (Conditional VaR / Tail VaR):** Measures the expected loss *conditional* on returns breaching the VaR cutoff, providing a coherent risk measure that captures tail severity during market crises such as March 2020 or the 2008 Global Financial Crisis. Under a normal distribution $r \sim N(\mu, \sigma^2)$, analytical Expected Shortfall at confidence level $1 - \alpha$ is given by:

$$\mathrm{ES}_{\alpha} = -\left[\mu - \sigma \frac{\phi(z_{\alpha})}{\alpha}\right]$$

Where: $\phi(z_{\alpha})$ is the standard normal probability density function evaluated at the critical quantile z_{α} (for $\alpha = 5\%$, $z_{0.05} = 1.645$ and $\phi(1.645)/\alpha = 2.063$).

Worked Example 4: Real Return, Parametric VaR, and Expected Shortfall

An Australian balanced institutional fund valued at $20,000,000 has an expected annual nominal return of $\mu = 8.40\%$ and annual volatility of $\sigma = 11.50\%$. Expected CPI inflation over the coming year is $i = 2.80\%$. Compute the expected real return and estimate the 1-year 95% parametric Value at Risk (VaR) and Expected Shortfall (ES) in both percentage and dollar terms.

1. **Compute the exact expected real return using the Fisher equation:**

$$r_{\text{real}} = \frac{1 + 0.0840}{1 + 0.0280} - 1 = \frac{1.0840}{1.0280} - 1 = 1.054475 - 1 = 5.45\%$$

Note that the simple approximation $8.40\% - 2.80\% = 5.60\%$ overstates the exact real return by 15 basis points.

2. **Calculate the 1-year 95% parametric VaR ($\alpha = 5\%, z_{0.05} = 1.645$):**

$$\mathrm{VaR}_{5\%} = -\left(0.0840 - 1.645 \times 0.1150\right) = -(0.0840 - 0.189175) = 10.52\%$$

$$\text{Dollar VaR}_{5\%} = 0.105175 \times \$20,000,000 = \$2,103,500$$

There is a 5% probability that the fund will suffer an annual loss exceeding 10.52% ($2,103,500) under normally distributed returns.

3. **Calculate the 1-year 95% Expected Shortfall (ES):**

$$\mathrm{ES}_{5\%} = -\left(0.0840 - 2.063 \times 0.1150\right) = -(0.0840 - 0.237245) = 15.32\%$$

$$\text{Dollar ES}_{5\%} = 0.153245 \times \$20,000,000 = \$3,064,900$$

Conditional on returns falling into the worst 5% tail, the expected loss is 15.32% ($3,064,900), highlighting why risk committees evaluate Expected Shortfall alongside VaR.

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Higher Moments: Skewness and Kurtosis in Financial Returns

Mean-variance analysis strictly describes portfolios when returns are jointly normally distributed or investors possess quadratic utility. Empirical equity and credit returns, however, frequently exhibit **negative skewness** (a longer left tail caused by sudden market crashes, liquidity freezes, and leveraged deleveraging) and **positive excess kurtosis** (leptokurtic distributions with fatter tails and sharper peaks than a Gaussian bell curve). Hedge fund strategies such as selling out-of-the-money index puts or merger arbitrage often display artificially high Sharpe ratios during calm markets precisely because they earn steady small premiums while embedding severe negative skewness that materialises only during systemic dislocations.

1.5 Risk Aversion, Utility Theory, and the Capital Allocation Line

Portfolio choice balances expected return against risk according to an investor's degree of **risk aversion**. Under mean-variance preferences, each portfolio with expected return $E(r)$ and variance σ^2 is assigned a utility score (also interpretable as its **Certainty Equivalent Rate**, the risk-free return an investor would view as equally desirable):

$$U = E(r) - \frac{1}{2} A \sigma^2$$

Where: U is the investor's utility (certainty equivalent rate, in decimals); $E(r)$ is the portfolio's expected return; σ^2 is the portfolio variance (volatility squared in decimals); and A is the coefficient of risk aversion (typically $A \in [2, 5]$ for moderately risk-averse investors, $A = 0$ for risk-neutral, and $A < 0$ for risk-seeking).

Plotting combinations of $(E(r), \sigma)$ that yield an identical utility level U produces an upward-sloping, convex **indifference curve**. More risk-averse investors (higher A) possess steeper indifference curves because they demand a larger incremental expected return for every unit of additional volatility. Rational investors seek to reach the highest attainable indifference curve (north-west direction in expected return–standard deviation space).

Combining a Risky Portfolio with the Risk-Free Asset

Suppose an investor allocates proportion y of total wealth to a risky portfolio P (with expected return $E(r_P)$ and standard deviation σ_P) and proportion $(1 - y)$ to the risk-free asset (paying r_f with $\sigma_f = 0$). The resulting **complete portfolio** C has expected return and standard deviation:

$$E(r_C) = r_f + y\,[E(r_P) - r_f], \qquad \sigma_C = |y|\sigma_P$$

Where: y is the portfolio weight in the risky portfolio P; $E(r_P) - r_f$ is the risk premium on portfolio P; and σ_C is the complete portfolio's standard deviation.

Eliminating $y = \sigma_C / \sigma_P$ (for $y \geq 0$) yields the linear equation of the **Capital Allocation Line (CAL)**, which traces every feasible risk-return combination achievable by mixing P and the risk-free asset:

$$E(r_C) = r_f + \underbrace{\left(\frac{E(r_P) - r_f}{\sigma_P}\right)}_{\text{Sharpe Ratio } (S_P)} \sigma_C$$

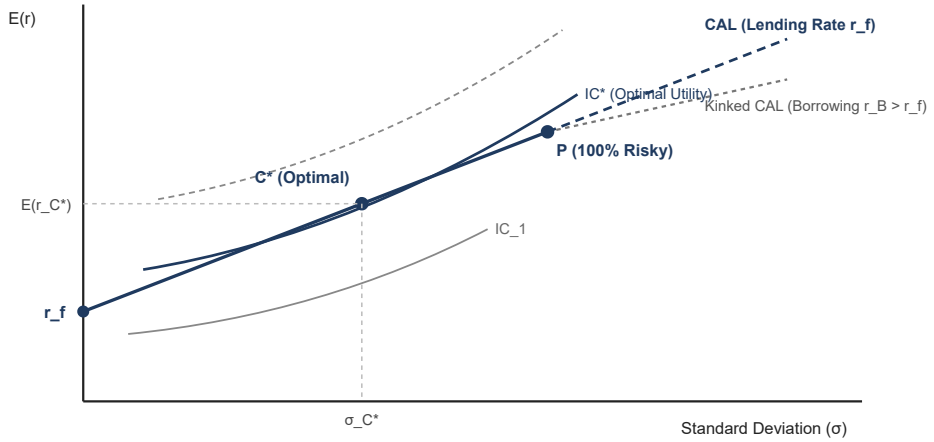
Where: the vertical intercept is the risk-free rate r_f; and the slope of the CAL is the **Sharpe Ratio** S_P, which measures incremental expected excess return earned per unit of portfolio volatility.

To find the optimal allocation y^* that maximises the investor's utility $U(y) = r_f + y[E(r_P) - r_f] - \frac{1}{2} A y^2 \sigma_P^2$, differentiate with respect to y and set the derivative to zero:

$$\frac{dU}{dy} = E(r_P) - r_f - Ay^*\sigma_P^2 = 0 \implies y^* = \frac{E(r_P) - r_f}{A\sigma_P^2}$$

Where: y^* is the utility-maximising fraction of wealth invested in the risky portfolio P; $E(r_P) - r_f$ is the risky portfolio's excess return; A is the investor's risk aversion coefficient; and σ_P^2 is the risky portfolio's variance.

Geometrically, y^* corresponds to the exact point of tangency where the highest feasible indifference curve touches the CAL. When $y^* < 1$, the investor lends $(1 - y^*)$ at the risk-free rate (e.g., holding cash or Treasury Notes); when $y^* > 1$, the investor borrows $(y^* - 1)$ to lever up the risky portfolio. If the investor's borrowing rate r_B exceeds the lending rate r_f, the CAL exhibits a **kink** at $y = 1$ (at portfolio P): for $y \leq 1$, the slope is $(E(r_P) - r_f)/\sigma_P$, whereas for $y > 1$ (levered positions), the slope flattens to $(E(r_P) - r_B)/\sigma_P$.



Worked Example 5: Optimal Capital Allocation and Utility Maximisation in AUD

An Australian superannuation member holds \$400,000 in retirement savings and exhibits a risk aversion coefficient of $A = 3.2$. The member can allocate between a diversified ASX/global risky portfolio P with expected return $E(r_P) = 10.60\%$ p.a. and standard deviation $\sigma_P = 16.00\%$ p.a., and a risk-free Australian Treasury Note yielding $r_f = 4.20\%$ p.a.

1. **Calculate the Sharpe Ratio of the risky portfolio P:**

$$S_P = \frac{E(r_P) - r_f}{\sigma_P} = \frac{0.1060 - 0.0420}{0.1600} = \frac{0.0640}{0.1600} = 0.4000$$

2. **Determine the optimal risky portfolio weight y^* and dollar allocations:**

$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.1060 - 0.0420}{3.2 \times (0.1600)^2} = \frac{0.0640}{3.2 \times 0.0256} = \frac{0.0640}{0.08192} = 0.78125 \text{ (or } 78.125\%)$$

   Thus, the member invests $0.78125 \times \$400,000 = \$312,500$ in the risky portfolio P and the remaining $0.21875 \times \$400,000 = \$87,500$ in the risk-free Treasury Note.

3. **Compute the expected return, volatility, Sharpe ratio, and utility of the optimal complete portfolio C^*:**

$$E(r_{C^*}) = 0.0420 + 0.78125 \times (0.1060 - 0.0420) = 0.0420 + 0.0500 = 9.20\%$$

$$\sigma_{C^*} = 0.78125 \times 16.00\% = 12.50\%$$

$$S_{C^*} = \frac{0.0920 - 0.0420}{0.1250} = 0.4000 \text{ (identical to } S_P\text{, as every point on the CAL shares the same slope)}$$

$$U(C^*) = 0.0920 - \frac{1}{2}(3.2)(0.1250)^2 = 0.0920 - 1.6(0.015625) = 0.0920 - 0.0250 = 6.70\%$$

Check: investing 100% in P yields $U(P) = 0.1060 - 1.6(0.0256) = 6.504\%$, and investing 100% in r_f yields $U(r_f) = 4.20\%$. Both are strictly inferior to $U(C^*) = 6.70\%$.

4. **Examine the effect of a higher borrowing rate ($r_B = 6.20\%$) for an aggressive investor with $A = 1.5$:** At $r_f = 4.20\%$, the unconstrained formula gives $y = 0.0640/(1.5 \times 0.0256) = 1.6667 > 1$, which requires borrowing. Recomputing with $r_B = 6.20\%$:

$$y^*_{\text{levered}} = \frac{0.1060 - 0.0620}{1.5 \times 0.0256} = \frac{0.0440}{0.0384} = 1.1458 \text{ (or } 114.58\%)$$

Because $y^*_{\text{levered}} > 1$, the aggressive investor borrows 14.58% of initial capital at 6.20% to hold 114.58% in P. Had the formula at r_B produced a weight below 1.0, the optimal choice would sit exactly at the kink $y^* = 1.0$.

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