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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Derivatives

Course Code: BFC2751 • Module 2 Notes

Module 2: Interest Rate & Currency Swaps, FRA Pricing & Australian Fixed-Income Derivatives

2.1 Australian Money Market Benchmarks, Day-Count Conventions & Term Structure

Fixed-income derivatives depend fundamentally on the benchmark interest rate curves used for projecting floating cash flows and discounting future payments. In the Australian financial system, three reference benchmarks anchor short-term and long-term interest rate derivative markets: the Reserve Bank of Australia (RBA) **Cash Rate**, the **Australian Overnight Index Average (AONIA)**, and the **Bank Bill Swap Rate (BBSW)**.

The RBA Cash Rate is the target interest rate for unsecured overnight interbank funds settled across Exchange Settlement Accounts at the RBA. **AONIA** is the transaction-based volume-weighted average rate of actual unsecured overnight cash market transactions, serving as Australia's near-risk-free overnight benchmark rate (analogous to SOFR in the United States or SONIA in the United Kingdom) and the floating leg of AUD Overnight Index Swaps (OIS). By contrast, **BBSW**—administered by ASX Benchmarks under ASIC's Financial Benchmark Rules—measures the rate at which prime Australian Authorised Deposit-taking Institutions (ADIs) issue and trade short-term bank-accepted bills and negotiable certificates of deposit across tenors from 1 month to 6 months. Because BBSW reflects prime bank credit and liquidity risk in addition to the risk-free rate, BBSW sits slightly above AONIA OIS rates of the same maturity, with the difference termed the **BBSW−OIS spread**.

Day-Count Conventions and Compounding Conversions

Accurate derivative pricing requires strict adherence to market day-count conventions when converting quoted annual rates into period cash flows:

- **Actual/365 (Fixed):** Standard in Australian money markets (BBSW, bank bills, AUD FRAs, and AUD floating swap legs). The accrual factor is $\tau = \text{Actual Days}/365$, regardless of leap years.
- **Actual/360:** Standard in US dollar and Euro money markets (SOFR, Euribor, USD FRAs). A quoted 90-day USD rate accrues over $90/360 = 0.25$ of a year.
- **Actual/Actual (ICMA / Bond Basis):** Used for Australian Commonwealth Government Bonds (ACGBs) and semi-annual fixed-rate Treasury bond coupons, where each semi-annual coupon period carries weight 0.5 between exact coupon dates.
- **30/360:** Common in US corporate bond markets, assuming 30 days per month and 360 days per year.

To convert between an interest rate R_m compounded m times per year and an equivalent continuously compounded rate r_c (or simple money-market rate R_s over maturity T), equivalence of terminal accumulation requires:

$$e^{r_c T} = \left(1 + \frac{R_m}{m}\right)^{mT} = 1 + R_s T \implies r_c = m \ln\left(1 + \frac{R_m}{m}\right) = \frac{\ln(1 + R_s T)}{T}$$

Where: r_c is the continuously compounded annual rate; R_m is the nominal annual rate compounded m times per year; R_s is the simple money-market interest rate; and T is the accrual period in years.

Zero-Coupon Rates, Discount Factors, and Implied Forward Rates

A **zero-coupon rate** (spot rate) $r(0,T)$ is the yield on a hypothetical single-payment security maturing at T with no intermediate coupons. The corresponding **discount factor** $P(0,T) = e^{-r_c(0,T)T}$ represents the present value of A$1 received with certainty at T. While short-dated zero rates up to 6 months are observed directly from cash and BBSW simple rates via $P(0,T) = 1/(1 + R_s T)$, longer-dated zero rates are extracted sequentially via **bootstrapping** from par swap rates or coupon bond prices: each maturity T_n is solved so that the discounted sum of coupons plus principal equals par.

Given two zero-coupon rates r_1 for maturity T_1 and r_2 for maturity $T_2 > T_1$ (both continuously compounded), no-arbitrage dictates an exact **implied forward interest rate** $r_F(T_1, T_2)$ for borrowing or lending between T_1 and T_2. Because investing A$1 at r_2 for T_2 must match investing at r_1 for T_1 and rolling over at r_F from T_1 to T_2, we obtain:

$$e^{r_2 T_2} = e^{r_1 T_1} e^{r_F (T_2 - T_1)} \implies r_F = \frac{r_2 T_2 - r_1 T_1}{T_2 - T_1}$$

In money-market simple-interest terms using discount factors $P(0,T_1)$ and $P(0,T_2)$ over accrual fraction $\tau = T_2 - T_1$, the simple implied forward rate R_F is:

$$1 + R_F \tau = \frac{P(0,T_1)}{P(0,T_2)} \implies R_F = \frac{1}{\tau}\left(\frac{P(0,T_1)}{P(0,T_2)} - 1\right)$$

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Where: r_1, r_2 are continuously compounded zero rates to T_1 and T_2; r_F is the continuously compounded forward rate over $[T_1, T_2]$; $P(0, T_i)$ is the discount factor to T_i; and R_F is the simple money-market forward rate for accrual τ.

Worked Example 1: Bootstrapping AUD Zero Rates and Deriving Implied Forward Rates

Suppose current Australian money-market simple BBSW rates (Actual/365 convention, approximated with exact quarterly fractions $T = 0.25, 0.50, 0.75, 1.00$ years) are 4.20% (3-month), 4.40% (6-month), 4.55% (9-month), and 4.68% (12-month). Compute the discount factors, continuously compounded zero rates, and simple 3-month implied forward rates for each consecutive quarter.

1. **Compute discount factors $P(0,T) = \frac{1}{1+R_s T}$ and continuous zero rates $r_c = -\frac{\ln P(0,T)}{T}$:**
   - $T = 0.25$: $P(0, 0.25) = \frac{1}{1+0.0420\times 0.25} = 0.989609$, $r_c(0, 0.25) = -\frac{\ln(0.989609)}{0.25} = 4.1781\%$.
   - $T = 0.50$: $P(0, 0.50) = \frac{1}{1+0.0440\times 0.50} = 0.978474$, $r_c(0, 0.50) = -\frac{\ln(0.978474)}{0.50} = 4.3523\%$.
   - $T = 0.75$: $P(0, 0.75) = \frac{1}{1+0.0455\times 0.75} = 0.967001$, $r_c(0, 0.75) = -\frac{\ln(0.967001)}{0.75} = 4.4741\%$.
   - $T = 1.00$: $P(0, 1.00) = \frac{1}{1+0.0468\times 1.00} = 0.955292$, $r_c(0, 1.00) = -\frac{\ln(0.955292)}{1.00} = 4.5738\%$.
2. **Derive simple 3-month implied forward rates ($\tau = 0.25$):**

$$R_F(0.25, 0.50) = \frac{1}{0.25}\left(\frac{0.989609}{0.978474} - 1\right) = 4.5522\%$$

$$R_F(0.50, 0.75) = \frac{1}{0.25}\left(\frac{0.978474}{0.967001} - 1\right) = 4.7456\%$$

$$R_F(0.75, 1.00) = \frac{1}{0.25}\left(\frac{0.967001}{0.955292} - 1\right) = 4.9027\%$$

Notice that because the spot curve is upward-sloping, each forward rate lies above the corresponding spot zero rate.

Maturity T (Years)	Simple Spot Rate R_s (%)	Discount Factor $P(0,T)$	Continuous Zero Rate r_c (%)	Forward Period	Implied Simple Forward Rate R_F (%)
0.25 (3M)	4.2000%	0.989609	4.1781%	0M × 3M (Spot)	4.2000%
0.50 (6M)	4.4000%	0.978474	4.3523%	3M × 6M	4.5522%
0.75 (9M)	4.5500%	0.967001	4.4741%	6M × 9M	4.7456%
1.00 (12M)	4.6800%	0.955292	4.5738%	9M × 12M	4.9027%

Multi-Year Swap Curve Bootstrapping, Duration and DV01

Beyond the 12-month money market, zero rates $r_c(0,T)$ are bootstrapped sequentially from liquid semi-annual par swap rates. Suppose 6-month and 12-month discount factors $P(0, 0.5)$ and $P(0, 1.0)$ are already known, and the 1.5-year par swap rate is $s_{1.5}$ (paid semi-annually with $\tau = 0.5$). Because a par swap prices its fixed bond leg at exactly 1.00 at initiation, the unknown discount factor $P(0, 1.5)$ solves:

$$\frac{s_{1.5}}{2}P(0,0.5) + \frac{s_{1.5}}{2}P(0,1.0) + \left(1 + \frac{s_{1.5}}{2}\right)P(0,1.5) = 1 \implies P(0,1.5) = \frac{1 - \frac{s_{1.5}}{2}\left[P(0,0.5) + P(0,1.0)\right]}{1 + \frac{s_{1.5}}{2}}$$

Where: $s_{1.5}$ is the quoted annual 1.5-year par swap rate; and $P(0, T_i)$ are the previously bootstrapped zero-coupon discount factors. Repeating this recursion out to 30 years constructs the complete AUD zero-coupon curve.

To measure interest-rate sensitivity of fixed-income instruments and swap books, analysts compute **Modified Duration (D_{mod})** and **Dollar Value of a Basis Point (DV01)**:

$$D_{\text{mac}} = \frac{1}{B}\sum_{i=1}^{n} t_i \cdot CF_i \cdot e^{-y t_i}, \qquad D_{\text{mod}} = \frac{D_{\text{mac}}}{1 + y/m}, \qquad \text{DV01} \approx D_{\text{mod}} \times B \times 0.0001$$

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Where: B is the bond's market value; CF_i is cash flow at time t_i; y is yield to maturity compounded m times per year; and DV01 measures the dollar price change for a 1-basis-point (0.01%) parallel shift in yield.

Continuing Worked Example 1, suppose the quoted 1.5-year AUD semi-annual par swap rate is $s_{1.5} = 4.75\%$ p.a., while $P(0, 0.5) = 0.978474$ and $P(0, 1.0) = 0.955292$. Substituting into the swap bootstrap recursion yields:

$$P(0, 1.5) = \frac{1 - 0.02375(0.978474 + 0.955292)}{1 + 0.02375} = \frac{1 - 0.045927}{1.02375} = 0.931939$$

Converting this discount factor to a continuously compounded 1.5-year zero rate gives $r_c(0, 1.5) = -\frac{\ln(0.931939)}{1.5} = 4.6992\%$ p.a. Institutional AUD swap desks repeat this bootstrap across 2-, 3-, 5-, 7-, 10-, 15-, and 30-year par swap quotes (interpolating missing semi-annual nodes via cubic splines or piecewise constant forward rates) to construct a smooth arbitrage-free AUD curve.

2.2 Forward Rate Agreements (FRAs) and ASX Interest Rate Futures

A **Forward Rate Agreement (FRA)** is an OTC forward contract in which two parties agree on a fixed interest rate R_K to apply to a notional principal L over a future reference period $[T_1, T_2]$. Standard market convention labels the contract an $M \times N$ FRA, where M is the number of months until the reference period begins (T_1) and N is the number of months until the reference period ends (T_2). For example, a 3×6 FRA covers a 3-month borrowing or lending period beginning in 3 months and ending in 6 months.

The **FRA buyer** (long position) agrees to *pay the fixed rate* R_K and *receive the floating reference rate* R_{ref} (such as 3-month BBSW observed at T_1). Corporate borrowers buy FRAs to hedge against rising interest rates prior to drawing down floating-rate bank bills. Conversely, the **FRA seller** (short position) agrees to *receive the fixed rate* R_K and *pay the floating rate* R_{ref}, protecting lenders or cash investors against falling yields.

FRA Valuation and Upfront Cash Settlement Mechanics

Although interest on the underlying loan would ordinarily be paid in arrears at T_2, market convention settles an FRA in cash at the *start* of the reference period (T_1) once R_{ref} is fixed. Consequently, the arrears interest differential $L(R_{\text{ref}} - R_K)\tau$ payable at T_2 is discounted back over $\tau = (T_2 - T_1)$ using the realised market rate R_{ref}. For the FRA buyer (pay-fixed, receive-floating), the settlement cash flow at T_1 is:

$$\text{Payoff}_{\text{buyer at } T_1} = L \cdot \frac{(R_{\text{ref}} - R_K)\tau}{1 + R_{\text{ref}}\tau}$$

Where: L is the notional principal; R_{ref} is the realised reference rate (e.g., 3M BBSW) observed at T_1; R_K is the contracted fixed FRA rate; and $\tau = \text{Days}/365$ (for AUD) is the accrual fraction for $[T_1, T_2]$.

At initiation $(t = 0)$, the fair fixed FRA rate R_K is set equal to the implied forward rate $R_F(T_1, T_2)$ so that the contract's initial market value is zero. During the life of an existing FRA prior to T_1, its mark-to-market value V_{FRA} to the buyer equals the present value of the difference between the current market forward rate R_F and the contracted rate R_K:

$$V_{\text{FRA}} = L \cdot (R_F - R_K)\tau \cdot P(0, T_2)$$

Where: R_F is the prevailing implied forward rate for $[T_1, T_2]$; R_K is the contract fixed rate; and $P(0, T_2)$ is the discount factor to T_2.

Worked Example 2: Pricing and Settling an AUD 3×6 FRA

A Sydney manufacturing company plans to borrow A$10,000,000 in 91 days for a subsequent 91-day working capital cycle (total horizon 182 days). To lock in its borrowing cost, the treasurer buys an AUD 3×6 FRA with notional $L = \$10,000,000$. Current spot BBSW quotes (Actual/365) are 4.20% for 91 days and 4.40% for 182 days. Determine the fair contracted FRA rate R_K. Then suppose 91 days later, 3-month BBSW resets at $R_{\text{ref}} = 4.95\%$. Compute the FRA settlement payment at T_1 and verify the company's effective borrowing rate.

1. **Compute the fair FRA rate R_K at $t = 0$:** Here $\tau_1 = 91/365 = 0.249315$, $\tau_2 = 182/365 = 0.498630$, and $\tau = 91/365 = 0.249315$.

$$1 + R_K\tau = \frac{1 + R_2\tau_2}{1 + R_1\tau_1} = \frac{1 + 0.0440 \times \frac{182}{365}}{1 + 0.0420 \times \frac{91}{365}} = \frac{1.021940}{1.010471} = 1.011350$$

$$R_K = \frac{0.011350}{91/365} = 0.045523 = 4.5523\% \text{ p.a.}$$

2. **Calculate the cash settlement at T_1 (day 91) when $R_{\text{ref}} = 4.95\%$:**

$$\text{Arrears Equivalent} = \$10,000,000 \times (0.0495 - 0.045523) \times \frac{91}{365} = \$9,914.48$$

Discounting to day 91 at $R_{\text{ref}} = 4.95\%$:

$$\text{Payoff}_{\text{buyer at } T_1} = \frac{\$9,914.48}{1 + 0.0495 \times \frac{91}{365}} = \frac{\$9,914.48}{1.012341} = \$9,793.61$$

Because BBSW rose above 4.5523%, the FRA seller pays A$9,793.61 to the manufacturing company at day 91.

3. **Verify the effective borrowing cost:** The company only needs to borrow $\$10,000,000 - \$9,793.61 = \$9,990,206.39$ in the spot market at 4.95%. At day 182, repayment equals $\$9,990,206.39 \times (1 + 0.0495 \times 91/365) = \$10,113,499.00$, which corresponds to an effective annualised rate of 4.5523% on the full A$10,000,000—matching R_K exactly.

ASX 24 Interest Rate Futures and Convexity Adjustment

Exchange-traded interest rate derivatives in Australia centre on three flagship ASX 24 contracts: the **90-Day Bank Accepted Bill (IR) Futures**, the **3-Year Commonwealth Treasury Bond Futures (YT)**, and the **10-Year Commonwealth Treasury Bond Futures (XT)**. All three employ an index quoting convention of $P_{\text{quote}} = 100 - y$, where y is the annualised percentage yield. Consequently, when interest rates rise, futures prices fall, and vice versa.

For the **ASX 90-Day Bank Accepted Bill Futures**, each contract represents A$1,000,000 face value of 90-day bills. Unlike US Eurodollar/SOFR futures where each basis point is fixed at US$25, the Australian contract is priced using exact present-value bill discounting on an Actual/365 basis:

$$V_{\text{bill}} = \frac{\$1,000,000}{1 + (\frac{100 - P_{\text{quote}}}{100}) \times \frac{90}{365}}$$

Where: V_{bill} is the AUD contract valuation; and P_{quote} is the ASX 24 quoted futures price (e.g., 95.60 implies 4.40% yield).

For the **3-Year and 10-Year Treasury Bond Futures**, the contract specifies a notional A$100,000 face-value Commonwealth Government Bond carrying a fixed 6.00% semi-annual coupon. Settlement is cash-settled against an ASX-selected basket of physical ACGBs with maturities near 3 or 10 years, valued via the standard bond annuity formula at the quoted yield $y = (100 - P_{\text{quote}})/100$. Specifically, the AUD contract value of a 10-Year Treasury Bond Futures contract ($N = 20$ semi-annual periods, $c = 3\%$ per half-year) is:

$$V_{10Y} = \$100,000\,[0.03\,(\frac{1 - (1 + y/2)^{-20}}{y/2}) + (1 + y/2)^{-20}]$$

Where: $y = (100 - P_{\text{quote}})/100$ is the annualised futures yield; and V_{10Y} is the AUD contract valuation. Notice that unlike US Treasury bond futures—which allow physical delivery with conversion factors and a cheapest-to-deliver (CTD) option—ASX bond futures are purely cash-settled against the average yield of the designated ACGB basket, eliminating CTD delivery squeezes.

When extracting forward rates from long-dated interest rate futures, analysts must account for the **convexity adjustment**. Because futures are marked-to-market daily whereas FRAs settle once at the reference period, a positive correlation between interest rates and reinvestment rates causes the futures-implied rate R_{futures} to exceed the true forward rate R_{forward}. Under the Hull-White short-rate approximation:

$$R_{\text{forward}} \approx R_{\text{futures}} - \frac{1}{2}\sigma^2 T_1 T_2$$

Where: σ is the annualised standard deviation of the short-term interest rate change (typically around 0.010 to 0.015); T_1 is the maturity of the futures contract in years; and T_2 is the maturity of the underlying deposit period ($T_1 + 0.25$ for 90-day bills).

Worked Example 4: Duration-Based Bond Portfolio Hedging with ASX 10-Year Treasury Bond Futures

An Australian fixed-income fund holds a A$30,000,000 portfolio of long-dated corporate and government bonds with a modified duration of $D_P = 7.40$ years. To protect the portfolio against an anticipated rise in yields over the next two months, the manager hedges using ASX 10-Year Commonwealth Treasury Bond Futures currently quoted at $P_{\text{quote}} = 95.80$ (implied yield $y = 4.20\%$ p.a. semi-annual). Compute the futures contract value, modified duration, and optimal number of contracts to short.

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1. **Compute the 10-Year Bond Futures contract value ($y/2 = 0.021$):**

$$V_{10Y} = \$100,000\,[0.03\,(\frac{1 - 1.021^{-20}}{0.021}) + 1.021^{-20}] = \$100,000\,[0.03(16.19129) + 0.660014] = \$114,575.26$$

2. **Compute the modified duration of the notional 6% 10-Year bond at 4.20% yield:** Evaluating the present-value weighted average cash-flow timing yields Macaulay duration $D_{\text{mac}} = 7.8397$ years, and modified duration:

$$D_F = \frac{7.8397}{1 + 0.042/2} = 7.6785 \text{ years}$$

3. **Compute the duration-based optimal number of futures contracts N^*:**

$$N^* = -\frac{V_P \times D_P}{V_F \times D_F} = -\frac{\$30,000,000 \times 7.40}{\$114,575.26 \times 7.6785} = -\frac{222,000,000}{879,766.13} = -252.34 \approx -252 \text{ contracts}$$

The fund shorts **252 ASX 10-Year Treasury Bond Futures contracts** to achieve duration neutrality.

Worked Example 3: Hedging a Corporate Bill Issuance with ASX 90-Day Bank Bill Futures

An ASX-listed infrastructure firm plans to issue A$20,000,000 face value of 90-day commercial bills in three months. To protect against rising short-term yields, the treasurer shorts 20 ASX 90-day Bank Bill futures contracts at $P_0 = 95.60$ (implied yield 4.40%). Three months later, spot 90-day BBSW rises to 5.10%, and the expiring futures contract settles at $P_T = 94.90$. Compute the futures profit and demonstrate how it offsets the firm's higher borrowing cost.

1. **Compute contract value at entry ($y_0 = 4.40\%$):**

$$V_0 = \frac{\$1,000,000}{1 + 0.0440 \times \frac{90}{365}} = \frac{\$1,000,000}{1.010849} = \$989,267.13$$

2. **Compute contract value at expiry ($y_T = 5.10\%$):**

$$V_T = \frac{\$1,000,000}{1 + 0.0510 \times \frac{90}{365}} = \frac{\$1,000,000}{1.012575} = \$987,580.83$$

3. **Calculate gain on 20 short futures contracts:**

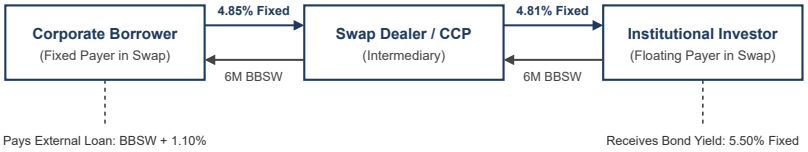
$$\text{Futures Gain} = 20 \times (\$989,267.13 - \$987,580.83) = 20 \times \$1,686.30 = \$33,725.93$$

4. **Compare against spot issuance proceeds:** At 4.40%, the firm would have raised $20 \times \$989,267.13 = \$19,785,342.60$. At 5.10%, physical bill proceeds drop to $20 \times \$987,580.83 = \$19,751,616.67$, a shortfall of $\$33,725.93$. Adding the $\$33,725.93$ futures gain restores net proceeds to exactly A$19,785,342.60 (an effective locked-in yield of 4.40%).

2.3 Plain-Vanilla Interest Rate Swaps (IRS): Mechanics & Comparative Advantage

A **plain-vanilla interest rate swap (IRS)** is an OTC agreement (or centrally cleared contract on ASX Clear (Futures) / LCH SwapClear) in which two counterparties exchange periodic interest payments on an agreed **notional principal** N in the same currency without exchanging the principal itself. One party—the **fixed-rate payer** (and floating-rate receiver)—pays a constant fixed annual rate s (the **swap rate**) at scheduled dates, while the **floating-rate payer** pays a variable market benchmark rate such as 3-month or 6-month BBSW (or compounded AONIA in an OIS). Because both legs are denominated in AUD and occur on matching dates, only the *net* cash difference is settled on each payment date.

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Market participants use swaps primarily to **transform liabilities or assets** without renegotiating underlying loan or bond indentures. A corporate borrower holding a floating-rate bank facility linked to BBSW can enter a pay-fixed/receive-floating swap to convert its debt into a synthetic fixed-rate liability. Similarly, an Australian superannuation fund holding fixed-rate corporate bonds can enter a pay-fixed/receive-floating swap to convert those fixed coupons into synthetic floating-rate returns, shortening portfolio duration. Australian banks (ADIs) extensively use AUD swaps to manage interest rate risk in the banking book (IRRBB) under APRA Prudential Standard APS 117.

Swap Market Conventions and Swap Spreads in Australia

In interdealer Australian swap markets, swap dealers quote two-way bid and ask fixed rates against 3-month BBSW (for maturities up to 3 years, paid quarterly) or 6-month BBSW (for maturities of 4 years and beyond, paid semi-annually), alongside AONIA OIS quotes. Dealers frequently express swap rates as a **swap spread** over the yield of a benchmark Australian Commonwealth Government Bond (ACGB) of matching maturity—or directly against ASX 3-Year and 10-Year Treasury Bond futures via an **Exchange for Physical (EFP)** transaction. A positive AUD swap spread reflects the net difference between bank credit/liquidity risk embedded in BBSW versus sovereign ACGB yields, as well as institutional supply-demand imbalances from corporate debt issuance and mortgage hedging.

The Comparative Advantage Argument and Quality Spread Differential (QSD)

Historically, one motivation for interest rate swaps rested on the **comparative advantage** of borrowers across fixed versus floating credit markets. Credit spreads demanded from lower-rated corporations over prime-rated corporations are typically wider in long-term fixed-rate bond markets than in short-term floating-rate bank loan markets (where lenders can reprice spreads or call loans periodically). Define the **Quality Spread Differential (QSD)** as the difference between the credit spread in the fixed-rate market and the credit spread in the floating-rate market:

$$\text{QSD} = (R_{\text{fixed, low}} - R_{\text{fixed, high}}) - (R_{\text{float, low}} - R_{\text{float, high}})$$

Where: $R_{\text{fixed, high}}$ and $R_{\text{fixed, low}}$ are the fixed borrowing rates of the higher-rated and lower-rated firms; and $R_{\text{float, high}}$ and $R_{\text{float, low}}$ are their floating spreads over BBSW. When $text{QSD} > o$, total borrowing cost across both firms can be reduced by QSD if each borrows in the market where it holds a comparative advantage and executes a swap.

Modern corporate finance notes an important critique of naive comparative advantage gains: the apparent arbitrage ignores **repricing and credit-rollover risk**. A lower-rated firm borrowing short-term floating funds at $\text{BBSW} + s$ faces the risk that its credit spread s will widen—or bank credit will be withdrawn—at future loan rollovers, whereas fixed-rate bond issuance locks in both the base rate and the credit spread for the entire term.

Worked Example 5: Designing a Comparative Advantage Interest Rate Swap

Two Australian companies—**Alpha Infrastructure Ltd (AA-rated)** and **Beta Logistics Pty Ltd (BBB-rated)**—each seek A$50,000,000 of 5-year funding. Alpha prefers floating-rate debt, whereas Beta prefers fixed-rate debt. Current borrowing rates available to each firm are shown below. An Australian swap bank proposes arranging a 5-year IRS for a total intermediary fee (bid-ask spread) of 0.08% p.a., splitting the remaining net benefit equally between Alpha and Beta. Design the swap and verify each party's final borrowing cost.

Borrower	Fixed-Rate Market Offer	Floating-Rate Market Offer	Preferred Liability
Alpha Infrastructure (AA)	5.20% p.a.	6M BBSW + 0.25%	Floating
Beta Logistics (BBB)	6.60% p.a.	6M BBSW + 1.05%	Fixed

Multi-Year Swap Curve Bootstrapping, Duration and DV01

Beyond the 12-month money market, zero rates $r_c(0,T)$ are bootstrapped sequentially from liquid semi-annual par swap rates. Suppose 6-month and 12-month discount factors $P(0,0.5)$ and $P(0,1.0)$ are already known, and the 1.5-year par swap rate is $s_{1.5}$ (paid semi-annually with $\tau = 0.5$). Because a par swap prices its fixed bond leg at exactly 1.00 at initiation, the unknown discount factor $P(0,1.5)$ solves:

$$\frac{s_{1.5}}{2}P(0,0.5) + \frac{s_{1.5}}{2}P(0,1.0) + (1+\frac{s_{1.5}}{2})P(0,1.5) = 1 \implies P(0,1.5) = \frac{1 - \frac{s_{1.5}}{2}\left[P(0,0.5) + P(0,1.0)\right]}{1 + \frac{s_{1.5}}{2}}$$

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Where: $s_{1.5}$ is the quoted annual 1.5-year par swap rate; and $P(0, T_i)$ are the previously bootstrapped zero-coupon discount factors. Repeating this recursion out to 30 years constructs the complete AUD zero-coupon curve.

To measure interest-rate sensitivity of fixed-income instruments and swap books, analysts compute **Modified Duration (D_{mod})** and **Dollar Value of a Basis Point (DV01)**:

$$D_{\text{mac}} = \frac{1}{B}\sum_{i=1}^{n} t_i \cdot CF_i \cdot e^{-y t_i}, \qquad D_{\text{mod}} = \frac{D_{\text{mac}}}{1 + y/m}, \qquad \text{DV01} \approx D_{\text{mod}} \times B \times 0.0001$$

Where: B is the bond's market value; CF_i is cash flow at time t_i; y is yield to maturity compounded m times per year; and DV01 measures the dollar price change for a 1-basis-point (0.01%) parallel shift in yield.

Continuing Worked Example 1, suppose the quoted 1.5-year AUD semi-annual par swap rate is $s_{1.5} = 4.75\%$ p.a., while $P(0, 0.5) = 0.978474$ and $P(0, 1.0) = 0.955292$. Substituting into the swap bootstrap recursion yields:

$$P(0, 1.5) = \frac{1 - 0.02375(0.978474 + 0.955292)}{1 + 0.02375} = \frac{1 - 0.045927}{1.02375} = 0.931939$$

Converting this discount factor to a continuously compounded 1.5-year zero rate gives $r_c(0, 1.5) = -\frac{\ln(0.931939)}{1.5} = 4.6992\%$ p.a. Institutional AUD swap desks repeat this bootstrap across 2-, 3-, 5-, 7-, 10-, 15-, and 30-year par swap quotes (interpolating missing semi-annual nodes via cubic splines or piecewise constant forward rates) to construct a smooth arbitrage-free AUD curve.

1. **Compute the Quality Spread Differential (QSD):**
   - Fixed-rate spread between Beta and Alpha: $6.60\% - 5.20\% = 1.40\%$.
   - Floating-rate spread between Beta and Alpha: $1.05\% - 0.25\% = 0.80\%$.
   - Total potential savings $\text{QSD} = 1.40\% - 0.80\% = 0.60\%$ p.a.
2. **Allocate total savings:** Subtracting the bank's 0.08% spread leaves $0.60\% - 0.08\% = 0.52\%$, or 0.26% net annual saving for each corporate. Thus:
   - Alpha's target floating cost = $(\text{BBSW} + 0.25\%) - 0.26\% = \text{BBSW} - 0.01\%$.
   - Beta's target fixed cost = $6.60\% - 0.26\% = 6.34\%$.
3. **Execute external borrowing in comparative-advantage markets:** Alpha holds an absolute advantage in both markets and a comparative advantage in fixed debt, so Alpha borrows externally at 5.20% **fixed**. Beta borrows externally at $\text{BBSW} + 1.05\%$ **floating**.
4. **Construct the swap legs with the intermediary bank:**
   - Swap 1 (Alpha ↔ Bank): Alpha pays **BBSW** to the bank and receives 5.21% **fixed** from the bank. Net cost for Alpha: $5.20\% + \text{BBSW} - 5.21\% = \text{BBSW} - 0.01\%$ (saving 0.26%).
   - Swap 2 (Bank ↔ Beta): Beta pays 5.29% **fixed** to the bank and receives **BBSW** from the bank. Net cost for Beta: $(\text{BBSW} + 1.05\%) + 5.29\% - \text{BBSW} = 6.34\%$ fixed (saving 0.26%).
   - Intermediary Bank Net Margin: Receives 5.29% fixed from Beta, pays 5.21% fixed to Alpha, and passes BBSW through, earning $5.29\% - 5.21\% = +0.08\%$ p.a. ($0.08\% \times \$50,000,000 = \$40,000$ per year).

2.4 Valuation of Interest Rate Swaps and Overnight Index Swaps (OIS)

At initiation, the fixed swap rate s is set so that the present value of the fixed leg equals the present value of the floating leg, giving the swap an initial value of zero. Over time, as interest rates shift, the swap acquires a positive mark-to-market value for one counterparty and an equal negative value for the other. Two mathematically equivalent methods value a plain-vanilla IRS: the **bond portfolio decomposition** and the **FRA portfolio decomposition**.

Method 1: Valuation as a Portfolio of Two Bonds

Adding a hypothetical exchange of principal N at maturity T_n leaves the net cash flows of a same-currency swap unchanged. Consequently, a swap that receives floating payments and pays fixed payments is identical to being **long a floating-rate bond** (B_{fl}) and **short a fixed-rate bond** (B_{fix}):

$$V_{\text{swap (pay-fix, rec-float)}} = B_{\text{fl}} - B_{\text{fix}}, \qquad V_{\text{swap (rec-fix, pay-float)}} = B_{\text{fix}} - B_{\text{fl}}$$

The fixed-rate bond value B_{fix} equals the discounted sum of remaining fixed coupons $k = N \cdot s \cdot \tau_i$ plus principal N at T_n. Under single-curve valuation (where discounting and projection use the same curve), a floating-rate bond resets to par N immediately after every coupon payment date. Therefore, at any intermediate date t between payment dates—when the next floating coupon $k^* = N \cdot R_{\text{reset}} \cdot \tau_1$ was already determined at

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the previous reset date—the floating bond's present value simplifies to the discounted sum of $N + k^*$ from the very next payment date T_1:

$$B_{\text{fl}} = (N + k^*)P(0, T_1), \qquad B_{\text{fix}} = \sum_{i=1}^{n} (N \cdot s \cdot \tau_i)P(0, T_i) + N \cdot P(0, T_n)$$

Where: N is the swap notional principal; s is the contracted annual fixed swap rate; k^* is the upcoming known floating payment determined at the last reset date; τ_i is the accrual fraction for period i; and $P(0, T_i)$ is the zero-coupon discount factor to payment date T_i.

Setting $B_{\text{fix}} = B_{\text{fl}} = N$ at swap initiation ($t = 0$) yields the closed-form formula for the **par swap rate** s_{par}:

$$s_{\text{par}} = \frac{1 - P(0, T_n)}{\sum_{i=1}^{n} \tau_i P(0, T_i)}$$

Method 2: Valuation as a Portfolio of FRAs and Dual-Curve OIS Discounting

Alternatively, an IRS can be viewed as a strip of sequential FRAs, one for each floating payment date T_i. Under the assumption that future floating rates will equal today's implied forward rates $R_{F,i}$, each future net exchange is valued directly at its forward rate and discounted to today:

$$V_{\text{swap (pay-fix, rec-float)}} = N \cdot (R_{\text{reset}} - s)\tau_1 P(0, T_1) + \sum_{i=2}^{n} N \cdot (R_{F,i} - s)\tau_i P(0, T_i)$$

Following the 2008 financial crisis, collateralised derivatives under ISDA CSAs earn interest at the overnight rate (AONIA in Australia). Consequently, dealers adopted **dual-curve valuation**: future floating cash flows are projected using forward BBSW rates extracted from the BBSW swap curve, while all cash flows (fixed and projected floating) are discounted using the **AONIA OIS zero curve**. Under dual-curve pricing, the FRA portfolio method remains exact without modification, whereas the simple par-reset property $B_{\text{fl}}(T_1) = N$ holds only on the projection curve.

Worked Example 6: Mid-Life Valuation of an AUD Interest Rate Swap (Dual Methods)

An Australian corporate treasurer holds a pay-fixed, receive-floating AUD interest rate swap with notional $N = \$25,000,000$ and a contracted fixed rate of $s = 4.60\%$ p.a. paid semi-annually ($\tau_i = 0.50$) against 6-month BBSW. The swap has 1.25 years remaining, with payment dates at $T_1 = 0.25$ years (3 months), $T_2 = 0.75$ years (9 months), and $T_3 = 1.25$ years (15 months). At the last reset date 3 months ago, 6-month BBSW was set at $R_{\text{reset}} = 4.40\%$ p.a. Current continuously compounded zero-coupon rates are 4.10% (0.25 yr), 4.25% (0.75 yr), and 4.40% (1.25 yr). Value the swap from the corporate's perspective using both the Bond Decomposition Method and the FRA Portfolio Method.

1. **Compute discount factors** $P(0, T_i) = e^{-r_i T_i}$**:**
   - $P(0, 0.25) = e^{-0.0410 \times 0.25} = 0.989802$
   - $P(0, 0.75) = e^{-0.0425 \times 0.75} = 0.968628$
   - $P(0, 1.25) = e^{-0.0440 \times 1.25} = 0.946485$
2. **Bond Decomposition Method:**
   - Semi-annual fixed coupon = $\$25,000,000 \times 0.0460 \times 0.50 = \$575,000.00$.

     $$B_{\text{fix}} = 575,000(0.989802) + 575,000(0.968628) + 25,575,000(0.946485) = \$25,332,454.91$$

   - Upcoming floating coupon $k^* = \$25,000,000 \times 0.0440 \times 0.50 = \$550,000.00$.

     $$B_{\text{fl}} = (\$25,000,000 + \$550,000) \times 0.989802 = \$25,289,450.10$$

   - Swap value (receive floating, pay fixed):

     $$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}} = \$25,289,450.10 - \$25,332,454.91 = -\$43,004.81$$

3. **FRA Portfolio Method (Verification):**
   - Implied forward rates for $[0.25, 0.75]$ and $[0.75, 1.25]$:

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$$R_{F,2} = \frac{1}{0.50}\left(\frac{0.989802}{0.968628} - 1\right) = 4.3721\%, \qquad R_{F,3} = \frac{1}{0.50}\left(\frac{0.968628}{0.946485} - 1\right) = 4.6789\%$$

- Present value of each net exchange $N(R_F - 0.0460)(0.50)P(0, T_i)$:
  - $T_1 = 0.25$: $\$25,000,000 \times (0.0440 - 0.0460) \times 0.50 \times 0.989802 = -\$24,745.06$
  - $T_2 = 0.75$: $\$25,000,000 \times (0.043721 - 0.0460) \times 0.50 \times 0.968628 = -\$27,593.42$
  - $T_3 = 1.25$: $\$25,000,000 \times (0.046789 - 0.0460) \times 0.50 \times 0.946485 = +\$9,333.67$
- Sum of FRA present values: $-\$24,745.06 - \$27,593.42 + \$9,333.67 = -\$43,004.81$. Both methods produce the identical valuation of $-\text{A}\$43,004.81$.

2.5 Currency Swaps and Cross-Currency Basis

A **cross-currency swap** exchanges principal and interest payments denominated in one currency (such as AUD) for principal and interest payments in another currency (such as USD or EUR). Unlike a single-currency IRS, currency swaps normally involve **three cash-flow stages**:

1. **Initial Principal Exchange ($t = 0$):** The two parties exchange principal amounts N_D (domestic currency) and N_F (foreign currency) calibrated at the prevailing spot exchange rate S_0 so that $N_D = S_0 N_F$.
2. **Periodic Interest Exchanges:** On scheduled dates, each party pays interest in the currency of the principal it received at $t = 0$. Because payments are in different currencies, they are generally not netted unless specified under an FX reset clause. Currency swaps may be structured as fixed-for-fixed, fixed-for-floating, or floating-for-floating (basis swaps).
3. **Final Principal Re-Exchange (T):** At maturity, the exact initial principal amounts N_D and N_F are re-exchanged at the *original* spot rate S_0, eliminating foreign exchange risk on principal redemption.

Cross-currency swaps play a central role in the Australian banking system. Because Australia runs a structural domestic savings-investment gap for bank wholesale funding, the four major Australian banks routinely issue USD, EUR, and GBP medium-term notes in offshore capital markets where investor demand is deep, and immediately execute **cross-currency basis swaps** to convert those foreign-currency debt proceeds and coupon obligations back into floating AUD (BBSW) liabilities. The **cross-currency basis spread** reflects relative funding supply and demand across currencies—commonly quoted as a spread (in basis points) added to the non-USD floating leg when swapped against flat USD SOFR.

At initiation of a par cross-currency swap, the principal amounts satisfy $N_D = S_0 N_F$ and each currency's fixed rate (if fixed-for-fixed) equals the domestic or foreign par swap rate so that $B_D = N_D$ and $B_F = N_F$, ensuring $V_{\text{swap}} = B_D - S_0 B_F = 0$. In floating-for-floating cross-currency basis swaps, however, strong Australian institutional demand to swap USD wholesale bond proceeds into AUD funding typically drives the AUD/USD cross-currency basis positive or negative depending on global dollar liquidity conditions—meaning an Australian bank paying USD SOFR and receiving 3M BBSW receives (or pays) an additional basis adjustment calibrated to supply and demand in cross-border funding markets.

Valuation of Currency Swaps

In the absence of default risk, a fixed-for-fixed currency swap can be valued either as a portfolio of two bonds denominated in separate currencies or as a strip of FX forward contracts. For a counterparty receiving domestic currency (AUD) cash flows and paying foreign currency (USD) cash flows, let B_D be the AUD present value of the domestic bond and B_F be the USD present value of the foreign bond. Converting B_F into AUD at the current spot exchange rate S_0 (quoted as AUD per 1 USD) gives:

$$V_{\text{currency swap (rec AUD, pay USD)}} = B_D - S_0 B_F$$

Where: B_D is the present value in domestic currency (AUD) of the domestic cash flows discounted at domestic zero rates; B_F is the present value in foreign currency (USD) of the foreign cash flows discounted at foreign zero rates; and S_0 is the current spot exchange rate (AUD per unit of foreign currency).

Worked Example 7: Valuing an Existing AUD/USD Fixed-for-Fixed Currency Swap

An Australian resource company holds an existing currency swap under which it receives 4.50% p.a. fixed in AUD on a notional of $N_{\text{AUD}} = \$15,000,000$ and pays 5.20% p.a. fixed in USD on a notional of $N_{\text{USD}} = \text{US}\$10,000,000$, with annual interest payments. The swap has exactly 2 years remaining (payments at $t = 1$ and $t = 2$, including principal re-exchange at $t = 2$). Currently, the spot exchange rate is $S_0 = 1.5400$ AUD/USD, and continuously compounded flat zero-coupon yield curves stand at $r_{\text{AUD}} = 4.20\%$ p.a. and $r_{\text{USD}} = 4.80\%$ p.a. Compute the current AUD mark-to-market value of this swap.

1. **Value the AUD bond leg (B_{AUD}):** Annual coupon = $\$15,000,000 \times 0.045 = \$675,000$.

$$B_{\text{AUD}} = \$675,000\, e^{-0.042\times 1} + \$15,675,000\, e^{-0.042\times 2}$$

$$B_{\text{AUD}} = \$675,000(0.958870) + \$15,675,000(0.919431) = \$647,237.13 + \$14,412,084.91 = \$15,059,322.04 \text{ AUD}$$

2. **Value the USD bond leg (B_{USD}):** Annual coupon = $\text{US}\$10,000,000 \times 0.052 = \text{US}\$520,000$.

$$B_{\text{USD}} = \text{US}\$520,000\, e^{-0.048\times 1} + \text{US}\$10,520,000\, e^{-0.048\times 2}$$

$$B_{\text{USD}} = \text{US}\$520,000(0.953134) + \text{US}\$10,520,000(0.908464) = \text{US}\$495,629.57 + \text{US}\$9,557,041.45 = \text{US}\$10,052,671.02$$

3. **Convert USD bond to AUD and compute swap value:**

$$S_0 B_{\text{USD}} = 1.5400 \times 10,052,671.02 = \$15,481,113.37 \text{ AUD}$$

$$V_{\text{swap}} = B_{\text{AUD}} - S_0 B_{\text{USD}} = \$15,059,322.04 - \$15,481,113.37 = -\$421,791.33 \text{ AUD}$$

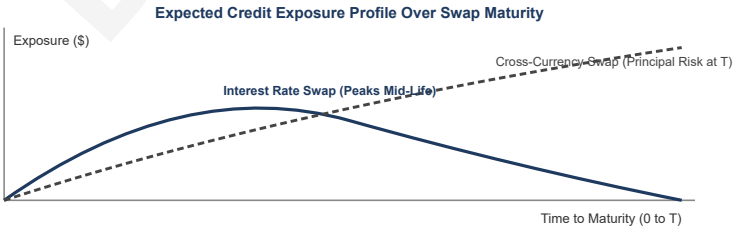
Because the USD appreciated relative to the original swap parity and USD fixed coupon rates exceed AUD rates, the swap is a liability worth A$421,791.33 to the Australian company.

Non-Standard Swap Structures and Counterparty Credit Exposure Profiles

Beyond plain-vanilla fixed-for-floating and fixed-for-fixed contracts, institutional fixed-income desks execute several specialised swap variants:

- **Tenor Basis Swaps:** Exchange two floating rates of different tenors within the same currency—such as 3-month BBSW against 6-month BBSW plus a quoted basis spread—allowing banks to manage funding-tenor mismatches.
- **Amortising and Accreting Swaps:** Contracts whose notional principal declines (matching project-finance or mortgage amortisation schedules) or increases over time according to a predetermined profile.
- **Forward-Starting Swaps and Swaptions:** A forward swap locks in a fixed swap rate today for a swap commencing at a future date T_1; a *swaption* grants the option (payer or receiver) to enter an IRS at a contracted strike swap rate, widely used to hedge callable corporate debt or mortgage prepayment risk.

From a counterparty credit risk perspective, single-currency interest rate swaps and cross-currency swaps display markedly different exposure trajectories over their lives. In an uncollateralised **interest rate swap**, expected credit exposure starts at zero, rises over the first third of the swap's life as interest rates diffuse away from inception levels, and then declines back toward zero as fewer coupon exchanges remain (*amortisation effect*). By contrast, in a **cross-currency swap**, the final exchange of principal at maturity T exposes counterparties to cumulative exchange-rate drift over the entire swap horizon, causing potential future exposure to remain high and peak near maturity.



2.6 Case Study: Multi-Currency Debt Restructuring and Cross-Currency Basis Spread Dynamics

In international treasury operations, Australian multinational corporations frequently issue foreign currency denominated bonds (such as US Dollar denominated Yankee bonds or Euro-denominated notes) to access deep, liquid capital markets with extended maturities. However, because their underlying revenue streams and functional currency are denominated in Australian Dollars (AUD), these entities face severe foreign exchange and interest rate volatility if the liabilities remain unhedged. A cross-currency interest rate swap (CCIRS) enables the corporate treasury to completely synthetically transform foreign currency debt into AUD liabilities, locking in predictable debt servicing costs.

The Cross-Currency Basis Spread Phenomenon

Under textbook covered interest parity (CIP), the forward exchange rate between two currencies is determined purely by the differential in their risk-free interest rates. In the post-2008 global financial architecture, however, persistent structural imbalances in the demand for US Dollar funding have caused covered interest parity to systematically fail. This persistent deviation is quoted in interbank markets as the *cross-currency*

basis spread.

When an Australian corporate borrower converts USD floating funding into AUD floating funding via a cross-currency swap, the borrower pays BBSW plus a spread that reflects both the borrower's credit standing and the negative cross-currency basis. A negative basis spread implies that market participants must pay a premium to borrow USD cash against foreign currency collateral, altering the effective synthetic funding cost of Australian corporates.

Financing Mechanism	Initial Cash Flow (t = 0)	Interim Debt Service (t = 1 to T-1)	Final Maturity Cash Flow (t = T)	Key Structural Risk Drivers
Domestic AUD Bank Facility	Receive AUD principal from domestic banking syndicate.	Pay quarterly 3-month BBSW + credit margin.	Repay AUD principal in full.	Domestic liquidity constraints; shorter available tenors (typically 3 to 5 years).
Unhedged US Dollar Yankee Bond	Receive USD principal from US institutional investors.	Pay semi-annual fixed USD coupon payments.	Repay full USD principal to bondholders.	Unhedged AUD/USD currency depreciation risk; severe balance sheet translation volatility.
Synthetic AUD via Fixed-to-Floating CCIRS	Exchange USD bond proceeds for AUD principal at spot exchange rate.	Receive USD fixed coupon from swap dealer; pay quarterly AUD BBSW + synthetic margin.	Exchange AUD principal back to USD at the original historical contract rate to retire bond.	Eliminates currency risk; replaces foreign exchange exposure with counterparty credit and basis risk.

Detailed Numerical Application: Synthetic AUD Debt Issuance

Consider **Melbourne Infrastructure Group (MIG)**, an ASX-listed infrastructure fund with revenue generated entirely in AUD. MIG successfully executes a US$500 million 5-year senior unsecured bond issue in the US debt market at a fixed coupon of 4.80% per annum, payable semi-annually. Concurrently, MIG enters into a 5-year cross-currency interest rate swap with an Australian major bank acting as swap counterparty.

The prevailing market rates at trade inception are as follows:

- Spot AUD/USD exchange rate: $S_0 = 0.6500$ (meaning $1 \text{ AUD} = 0.6500 \text{ USD}$).
- MIG converts US$500,000,000 into Australian Dollars at inception:

$$\text{AUD Principal} = \frac{\text{USD Principal}}{S_0} = \frac{500{,}000{,}000}{0.6500} = 769{,}230{,}769.23 \text{ AUD}$$

Where: USD Principal is the foreign currency debt raised, S_0 is the spot exchange rate at inception, and AUD Principal is the synthetic AUD cash received by MIG.

The swap agreement establishes the following matched obligations:

1. **At Settlement (t = 0):** MIG delivers the US$500 million bond proceeds to the swap bank and receives A$769,230,769.23 from the swap bank to fund its domestic Victorian capital expenditure projects.
2. **Interim Cash Flows (Semi-annual):**
   - Swap bank pays MIG fixed USD 4.80% per annum on US$500 million: $500{,}000{,}000 \times (0.0480/2) = 12{,}000{,}000$ USD. MIG immediately passes this US$12 million to US bondholders, completely covering its foreign debt obligations.
   - MIG pays the swap bank floating AUD interest based on 6-month BBSW + 140 basis points on the agreed A$769,230,769.23 principal.
3. **At Final Maturity (t = 5 years):**
   - Swap bank delivers US$500,000,000 to MIG. MIG utilizes this cash to redeem the maturing bond from US institutional investors.
   - MIG delivers A$769,230,769.23 to the swap bank, perfectly unwinding the transaction at the contractual initial spot rate, regardless of where the AUD/USD market exchange rate is trading in year 5.

Through this contractual derivative architecture, MIG secures access to global fixed-income institutional investors while transforming its foreign debt service into domestic floating obligations that correlate naturally with its inflation-linked infrastructure returns.

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