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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Derivatives

Course Code: BFC2751 • Module 1 Notes

Module 1: Derivative Markets, Forwards & Futures Mechanics, Cost-of-Carry Pricing & Hedging

1.1 Nature of Derivatives and Australian Market Architecture

A **derivative** is a financial instrument whose value is derived from the price or value of an underlying reference variable. Underlying variables commonly include equities, equity indices (such as the S&P/ASX 200), debt securities, interest rates (such as the Bank Bill Swap Rate, BBSW), foreign exchange rates (such as AUD/USD), commodities (iron ore, thermal coal, electricity, wheat, and crude oil), and credit events. Unlike primary securities such as ordinary shares or Commonwealth Government Bonds, derivatives do not represent direct claims on real corporate assets or new capital formation; rather, they reallocate price, interest-rate, and currency risks among market participants.

From a structural perspective, derivative instruments fall into two broad classes. **Forward commitments**—comprising forward contracts, futures contracts, and swaps—obligate both counterparties to transact at a predetermined price or formula on a specified future date. **Contingent claims**—principally options—confer the right, without the obligation, to buy or sell an underlying asset at an agreed strike price, creating asymmetric payoff profiles in exchange for an upfront premium.

Exchange-Traded versus Over-the-Counter (OTC) Markets in Australia

Derivatives trade across two distinct institutional venues: organised exchanges and over-the-counter (OTC) dealer networks. In Australia, exchange-traded derivatives are listed primarily on the **Australian Securities Exchange (ASX)** across two clearing platforms: **ASX 24** (formerly the Sydney Futures Exchange), which hosts equity index futures (SPI 200), Australian interest rate futures (90-Day Bank Accepted Bills, 3-Year and 10-Year Commonwealth Treasury Bonds), and energy/agricultural futures; and the **ASX Equity Derivatives Market**, which trades single-stock and index exchange-traded options (ETOs).

In the OTC market, banks, institutional asset managers, corporate treasuries, and commodity producers negotiate bilateral contracts tailored to specific notional amounts, maturities, and settlement dates. Australian OTC documentation generally follows the **International Swaps and Derivatives Association (ISDA) Master Agreement**, supplemented by a Credit Support Annex (CSA) governing bilateral collateral exchange, alongside Australian Financial Markets Association (AFMA) market conventions. Following the 2008 Global Financial Crisis and G20 Pittsburgh commitments, Australia enacted mandatory central clearing and trade reporting reforms under Part 7.5A of the *Corporations Act 2001 (Cth)*, supervised by the Australian Securities and Investments Commission (ASIC) and the Reserve Bank of Australia (RBA). Standardised AUD interest rate swaps and major G4 swaps must now be cleared through licensed central counterparties (CCPs) such as **ASX Clear (Futures)** or **LCH SwapClear**.

Institutional Attribute	Exchange-Traded Derivatives (ASX 24 / ASX)	Over-the-Counter (OTC) Derivatives
Contract Terms	Fully standardised contract size, tick size, delivery months, and settlement rules.	Customised notional principal, maturity dates, underlying grade, and payoff formulas.
Counterparty Credit Risk	Eliminated bilaterally via novation to a central counterparty (ASX Clear / ASX Clear (Futures)).	Bilateral credit exposure unless centrally cleared or collateralised under an ISDA CSA.
Margining & Cash Flows	Initial margin plus daily mark-to-market variation margin settled in cash.	Traditionally settled at maturity; post-APRA CPS 226 requires initial and variation margin for large entities.
Liquidity & Price Transparency	High pre-trade and post-trade transparency via central limit order book (ASX Trade24).	Dealer-quoted bid-ask spreads; post-trade reporting to ASIC-licensed trade repositories.
Closing Out Positions	Reversing (offsetting) an identical opposite contract on exchange prior to expiry.	Requires bilateral termination agreement, novation, or entering an offsetting trade with exposure remaining.

Market Participants: Hedgers, Speculators, and Arbitrageurs

Liquidity and price discovery in derivative markets rely on the interaction of three functional classes of participants:

- **Hedgers:** Non-financial corporations, agricultural exporters, mining firms, superannuation funds, and Authorised Deposit-taking Institutions (ADIs) that use derivatives to reduce or transform pre-existing exposures to commodity prices, exchange rates, equity market beta, or interest rates. An Australian gold producer selling forward contracts locks in an AUD sales price, sacrificing upside windfalls in exchange for cash-flow certainty.
- **Speculators:** Traders who take directional or volatility positions without an offsetting physical exposure, expecting the future spot price to diverge from the current forward or futures price. Speculators absorb the net imbalance of hedgers' orders and provide continuous market depth.
- **Arbitrageurs:** Quantitative dealers and proprietary desks that simultaneously execute offsetting transactions across spot and derivative markets (or across related maturities) to lock in a riskless profit whenever prices violate no-arbitrage bounds. Arbitrage activity enforces the law of one price and anchors forward prices to spot prices and financing costs.

Clearing House Risk Waterfall and Short-Selling Mechanics

Central counterparties such as **ASX Clear (Futures)** eliminate bilateral counterparty default risk by interposing themselves as buyer to every seller and seller to every buyer immediately after trade execution (*novation*). To survive extreme market shocks without taxpayer bailouts, ASX Clear (Futures) maintains a multi-layered **default waterfall**:

1. **Defaulter's Initial and Variation Margin:** The defaulting clearing participant's posted collateral is liquidated first to cover close-out losses.
2. **Defaulter's Default Fund Contribution:** Additional mutualised capital committed by the failed participant.
3. **CCP Skin-in-the-Game:** ASX Clear (Futures) commits a dedicated tranche of its own shareholder capital ahead of surviving members' contributions, aligning risk-management incentives.
4. **Mutualised Clearing Participant Default Fund & Recovery Powers:** If losses exceed the CCP tranche, surviving clearing participants' mutualised default fund contributions absorb residual shortfalls, supported by statutory assessment powers under ASX Operating Rules.

In arbitrage and hedging operations, executing reverse cash-and-carry trades requires **short selling** the underlying asset. Under ASIC Regulatory Guide 196 and ASX Market Integrity Rules, covered short selling requires borrowing the stock through a securities lending agreement (posting cash or government bonds as collateral) prior to executing the short sale, paying a borrowing fee (typically 0.25% to 1.00% p.a. for liquid S&P/ASX 200 shares, or higher for "special" hard-to-borrow stocks), and reimbursing the stock lender for any gross cash dividends declared during the loan period. For Commonwealth Government Bonds, arbitrageurs finance long or short positions via **repurchase agreements (repos)** at the general collateral (GC) or special repo rate.

1.2 Mechanics of Forward and Futures Contracts

A **forward contract** is a binding bilateral agreement to buy (*long position*) or sell (*short position*) a specified quantity of an underlying asset at a fixed delivery price K on a designated future maturity date T. At initiation ($t = 0$), the delivery price K is set equal to the prevailing forward price F_0 so that the contract has zero initial market value and neither party pays a premium to enter the agreement.

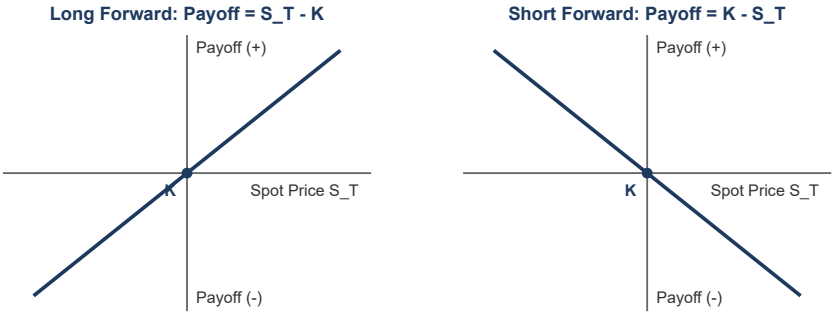
At maturity T, if the prevailing spot price of the underlying asset is S_T, the terminal payoffs per unit of underlying asset for the long and short forward positions are:

$$\text{Payoff}_{\text{long}} = S_T - K, \qquad \text{Payoff}_{\text{short}} = K - S_T$$

Where: S_T is the spot price of the underlying asset at contract maturity T; and K is the contracted delivery price agreed at initiation.

Because forward and futures payoffs are linear in S_T, the contract is a zero-sum game before transaction costs: every dollar gained by the long counterparty equals one dollar lost by the short counterparty.

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Institutional Specifications of ASX 24 Futures Contracts

A **futures contract** has the same economic objective as a forward contract—locking in a future transaction price—but is standardised and traded on an exchange. On ASX 24, every contract specification fixes five core parameters:

1. **Underlying Asset and Quality Grade:** For commodities (e.g., ASX Eastern Australia Wheat), the exchange defines acceptable deliverable grades and port zones; for financial futures such as the **ASX SPI 200 Index Futures**, the underlying is the S&P/ASX 200 equity index.
2. **Contract Size and Multiplier:** The SPI 200 contract carries a multiplier of A$25 per index point. If the SPI 200 futures price is $7,800$ points, one contract controls a notional equity exposure of $7,800 \times \$25 = \$195,000$.
3. **Minimum Tick Size:** The smallest permitted price increment. For SPI 200 futures, the minimum tick is 1 index point (valued at A$25 per contract). For 90-day Bank Accepted Bill futures, the tick is 0.01% (1 basis point) on a A$1,000,000 90-day face value.
4. **Delivery Months and Last Trading Day:** Financial futures on ASX 24 normally follow the quarterly March, June, September, and December cycle (expiry on the third Thursday of the contract month for SPI 200).
5. **Settlement Mechanism:** Index and interest rate futures such as SPI 200 and 90-day BBSW futures are **cash-settled** against the official Special Opening Quotation (SOQ) or ASX Benchmark closing rate, eliminating physical delivery logistics. Commodity and certain bond contracts permit physical delivery or Exchange for Physical (EFP) mechanisms.

Two market activity metrics are closely monitored by analysts. **Trading volume** counts the total number of contracts traded during a session, whereas **open interest** measures the total number of active, unclosed contracts outstanding at the end of the day. When a new buyer and a new seller trade, open interest rises by one; when an existing long and an existing short close out their positions, open interest falls by one; when one trader opens a new position against another trader closing an existing position, open interest stays unchanged. As a futures contract approaches its delivery period, arbitrage forces the futures price F_t to converge toward the spot price S_t so that $F_T = S_T$ at expiry. If $F_T > S_T$ during delivery, an arbitrageur would buy the spot asset, short the futures contract, and immediately make delivery to earn $F_T - S_T$.

Daily Marking-to-Market and Clearing House Margining

Unlike a forward contract, where cumulative gains and losses are realised only at maturity T, a futures contract is settled daily through **marking-to-market (MTM)**. When a trader opens a position through an ASX clearing participant, they must deposit an **initial margin (IM)**—a performance bond calibrated by ASX Clear (Futures) using Risk-Based Margining (SPAN or Historical VaR) to cover potential worst-case losses over a one-to-two-day liquidation horizon. At the close of each trading day, ASX Clear (Futures) computes the official daily settlement price and transfers **variation margin (VM)** from losing accounts to winning accounts.

If cumulative losses reduce a trader's margin balance below the specified **maintenance margin (MM)** threshold, the broker issues a **margin call** requiring the trader to deposit sufficient cash (variation margin) to restore the account all the way back up to the *initial margin* level—not merely to the maintenance margin. Failure to meet a margin call within the stipulated deadline empowers the broker to liquidate the futures position immediately. Excess balances above the initial margin may be withdrawn by the trader or earn interest on cash/approved collateral (such as Commonwealth Government Securities).

Worked Example 1: Daily Marking-to-Market Margin Account on ASX SPI 200 Futures

A Melbourne institutional portfolio manager takes a **long position in 4 ASX SPI 200 Index Futures** contracts on Day 0 at a trade price of $F_0 = 7,820$ index points. Each contract has a multiplier of A$25 per index point. The broker requires an initial margin of A$12,500 per contract (A$50,000 total) and a maintenance margin of A$9,500 per contract (A$38,000 total).

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Over the next four trading days, the daily settlement prices are $7,785$, $7,690$, $7,730$, and $7,850$. Track the daily cash flows, margin calls, and cumulative profit or loss.

1. **Day 0 (Initiation):** Notional exposure equals $4 \times 7,820 \times \$25 = \$782,000$. The manager deposits the initial margin of $\$50,000$.
2. **Day 1 (Settlement at 7,785):** The index falls by 35 points. Daily MTM loss is $4 \times \$25 \times (7,785 - 7,820) = -\$3,500$. The margin balance drops to $\$50,000 - \$3,500 = \$46,500$. Since $\$46,500 \geq \$38,000$, no margin call is triggered.
3. **Day 2 (Settlement at 7,690):** The index falls a further 95 points. Daily MTM loss is $4 \times \$25 \times (7,690 - 7,785) = -\$9,500$. The margin balance falls to $\$46,500 - \$9,500 = \$37,000$. Because *$37,000 < $38,000* (maintenance threshold breached), a margin call is issued to restore the account to the initial margin of $\$50,000$. Required deposit = $\$50,000 - \$37,000 = \$13,000$.
4. **Day 3 (Settlement at 7,730):** The index rises 40 points. Daily MTM gain is $4 \times \$25 \times (7,730 - 7,690) = +\$4,000$. Balance increases to $\$50,000 + \$4,000 = \$54,000$.
5. **Day 4 (Settlement at 7,850 and Close-out):** The index rises 120 points. Daily MTM gain is $4 \times \$25 \times (7,850 - 7,730) = +\$12,000$. Ending balance reaches $\$54,000 + \$12,000 = \$66,000$.

Trading Day	Settlement Price (pts)	Price Change (pts)	Daily MTM Gain / Loss (A$)	Cumulative MTM PnL (A$)	Margin Balance Pre-Call (A$)	Margin Call Deposit (A$)	Ending Margin Balance (A$)
Day 0 (Open)	7,820	—	—	0.00	50,000.00	0.00	50,000.00
Day 1	7,785	-35	-3,500.00	-3,500.00	46,500.00	0.00	46,500.00
Day 2	7,690	-95	-9,500.00	-13,000.00	37,000.00	13,000.00	50,000.00
Day 3	7,730	+40	+4,000.00	-9,000.00	54,000.00	0.00	54,000.00
Day 4 (Close)	7,850	+120	+12,000.00	+3,000.00	66,000.00	0.00	66,000.00

Verification: Total cash deposited by the fund equals $\$50,000 + \$13,000 = \$63,000$, and the ending balance is $\$66,000$, yielding a net profit of $\$66,000 - \$63,000 = \$3,000$, which matches $4 \times \$25 \times (7,850 - 7,820) = \$3,000$.

1.3 Cost-of-Carry Pricing and No-Arbitrage Valuation

The fair forward price F_0 of an asset is determined not by subjective expectations of the future spot price $E[S_T]$, but by **no-arbitrage replication**. Under the standard frictionless market assumptions—zero transaction costs, identical risk-free borrowing and lending rates r (expressed with continuous compounding), unrestricted short selling of assets, and zero credit risk—any asset that is held primarily for investment purposes must satisfy an exact cost-of-carry parity relationship.

Investment Asset with No Income

Consider a non-dividend-paying share trading at spot price S_0, with a forward contract maturing in T years when the continuously compounded risk-free rate is r. The unique no-arbitrage forward price is:

$$F_0 = S_0 e^{rT}$$

Where: F_0 is the fair forward price at $t = 0$; S_0 is the current spot price of the underlying asset; r is the continuously compounded risk-free interest rate per annum for maturity T; and T is the time to maturity in years.

If *F_0 > S_0 e^{rT}*, an arbitrageur executes a **cash-and-carry arbitrage**: borrow S_0 at rate r, buy one unit of the spot asset, and short one forward contract at F_0. At T, the asset is delivered for F_0 and the loan repayment is $S_0 e^{rT}$, locking in a riskless profit of *F_0 - S_0 e^{rT} > 0*. Conversely, if *F_0 < S_0 e^{rT}*, an arbitrageur executes a **reverse cash-and-carry**

arbitrage: short-sell the spot asset, invest the proceeds S_0 at rate r, and go long the forward contract at F_0, earning $S_o e^{rT} - F_o > o$ at T. Under discrete annual compounding at rate R, the equivalent relationship is $F_0 = S_0(1+R)^T$.

Investment Asset with Known Cash Income or Continuous Yield

When an underlying equity or bond pays known discrete cash flows (such as dividends or coupons) prior to T with total present value I discounted at the risk-free rate, holding the spot asset reduces the net financing cost by I. The no-arbitrage forward price becomes:

$$F_0 = (S_0 - I)e^{rT}, \qquad I = \sum_{j=1}^{k} D_j e^{-rt_j}$$

Where: I is the present value at $t = 0$ of all known cash income payments D_j occurring at times $t_j \in (0, T]$; S_0 is the current spot price; r is the continuously compounded risk-free rate; and T is the contract maturity in years.

For broad equity indices (such as the S&P/ASX 200) where constituent dividends arrive frequently throughout the year, dividends are modelled as a continuous proportional yield q per annum. Investing $S_0 e^{-qT}$ at $t = 0$ with dividends continuously reinvested in the index grows to exactly one unit of the index at T. Thus, the fair index futures price is:

$$F_0 = S_0 e^{(r-q)T}$$

Where: q is the continuously compounded annual dividend yield on the index; r is the continuously compounded domestic risk-free rate; and S_0 is the current index level.

The same continuous-yield architecture governs foreign exchange forward rates via **Covered Interest Rate Parity (CIRP)**. When the spot exchange rate S_0 is quoted as units of domestic currency per one unit of foreign currency (for example, AUD per 1 USD), holding one unit of foreign currency earns the foreign risk-free rate r_f, which plays the exact mathematical role of a continuous dividend yield q. Accordingly, the fair FX forward rate is:

$$F_0 = S_0 e^{(r_d - r_f)T}$$

Where: S_0 is the spot exchange rate (domestic currency per unit of foreign currency); r_d is the continuously compounded domestic risk-free rate (e.g., AUD); r_f is the continuously compounded foreign risk-free rate (e.g., USD); and T is the maturity in years.

Consumption Assets, Storage Costs, and Convenience Yield

Commodities held primarily for consumption—such as natural gas, copper, wheat, or thermal coal—incur physical storage, insurance, and warehousing costs. Let U denote the present value of net storage costs over $[0, T]$, or u denote the proportional continuous storage cost rate per annum. Unlike financial assets, industrial users hold physical commodity inventories to avoid production disruptions and stock-outs, conferring a non-monetary flow benefit called the **convenience yield** (y). Furthermore, because consumption commodities cannot always be borrowed freely in the repo market for short selling, reverse cash-and-carry arbitrage is constrained. Consequently, no-arbitrage yields an upper bound inequality and a convenience-yield pricing identity:

$$F_0 \leq (S_0 + U)e^{rT}, \qquad F_0 = S_0 e^{(r+u-y)T}$$

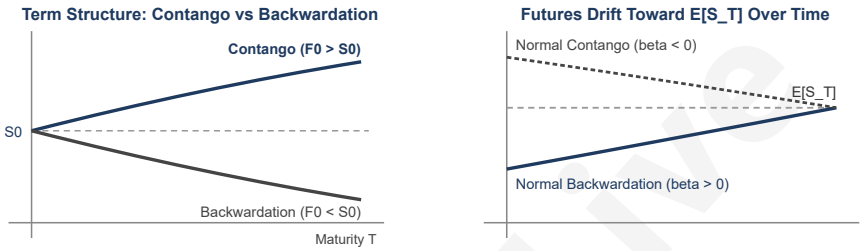
Where: U is the present value of fixed storage costs over $[0, T]$; u is the annual proportional storage cost rate; and y is the continuously compounded convenience yield reflecting the benefit of physical inventory ownership.

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When $F_o > S_o$ (typical when $r + u > y$, such as abundant storage), the futures curve is in **contango** (normal carry). When scarcity drives the convenience yield high enough that $y > r + u$, $F_o < S_o$ and the futures curve is in **backwardation**. The unified cost-of-carry rate c across all asset classes is defined such that $F_0 = S_0 e^{cT}$, where $c = r$ (non-dividend stock), $c = r - q$ (index/dividend stock), $c = r_d - r_f$ (FX), and $c = r + u - y$ (consumption commodity).

Futures Prices versus Expected Future Spot Prices: Contango, Normal Backwardation & Index Arbitrage

Students frequently confuse the shape of the futures term structure relative to today's spot price S_0 with the relationship between today's futures price F_0 and the *expected future spot price* $E[S_T]$. Under the **Expectations Hypothesis**, $F_0 = E[S_T]$ only if market participants are risk-neutral or if spot price shocks carry zero systematic risk. By contrast, Keynes and Hicks formulated the theory of **Normal Backwardation**, arguing that when commercial producers (such as Australian mining or wheat growers) dominate hedging as net short hedgers, speculators require a positive risk premium to assume long futures exposure—forcing *F_o < E[S_T]* so futures prices drift upward toward $E[S_T]$ over time. Modern asset pricing links this wedge to systematic risk via the Capital Asset Pricing Model (CAPM): if the underlying asset has positive beta (*beta > o*, as in equities or industrial metals), investors demand a risk premium, implying *F_o < E[S_T]*; if $\beta = 0$ (e.g., uncorrelated commodities), $F_0 = E[S_T]$; and if *beta < o*, *F_o > E[S_T]*.



In equity index markets, **index arbitrage** keeps ASX SPI 200 futures aligned with the underlying S&P/ASX 200 cash basket. Because trading all 200 constituent ASX shares simultaneously incurs brokerage commissions, bid-ask spreads, and market impact costs c_{tc}, exact equality $F_0 = S_0 e^{(r-q)T}$ is replaced in practice by a **no-arbitrage transaction-cost band**:

$$S_0 e^{(r-q)T} - c_{\text{tc}} \le F_0 \le S_0 e^{(r-q)T} + c_{\text{tc}}$$

Where: c_{tc} denotes total round-trip spot-basket transaction and short-borrowing costs expressed in index points. Whenever F_0 breaches the upper or lower boundary, institutional program-trading desks execute automated basket orders on the ASX against offsetting SPI 200 futures contracts until parity is restored.

Valuing an Existing Forward Contract During Its Life and Forward-Futures Price Differences

Although a forward contract has zero value at initiation ($f_0 = 0$ when $K = F_0$), subsequent movements in the spot price and interest rates alter its mark-to-market value f_t at any intermediate date $t \in (0, T)$. By comparing the existing contract (delivery price K) against a newly initiated forward contract with delivery price F_t expiring at the same date T, a long position holder effectively locks in a terminal cash difference of $F_t - K$ at T. Discounting back to time t yields the exact value of a long forward contract:

$$f_t = (F_t - K)e^{-r(T-t)}$$

Equivalently, substituting the cost-of-carry expression for F_t over remaining maturity $\tau = T - t$ gives $f_t = S_t - I_t - Ke^{-r\tau}$ (discrete income), $f_t = S_t e^{-q\tau} - Ke^{-r\tau}$ (continuous yield q), and $f_t = S_t e^{-r_f\tau} - Ke^{-r_d\tau}$ (foreign exchange). For a short forward position, the value is $-f_t = (K - F_t)e^{-r(T-t)}$.

When interest rates are deterministic, forward and futures prices for identical maturities are mathematically equal. When interest rates fluctuate stochastically, however, daily marking-to-market creates a slight divergence (the **convexity bias**). If the underlying asset price is positively correlated with interest rates, a long futures trader receives positive variation margin precisely when rates are high (reinvesting gains at higher rates) and pays margin when rates are low (financing losses at lower

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rates). Consequently, the futures price slightly exceeds the forward price when *text{Corr}(S, r) > 0*, and falls slightly below the forward price when *text{Corr}(S, r) < 0*. This effect is negligible for short-dated equity contracts but material for long-dated interest rate futures.

Worked Example 2: Cash-and-Carry Arbitrage on an ASX-Listed Share

Shares of an ASX-listed resources company trade at $S_0 = \$84.00$. The company will pay a single cash dividend of $D = \$2.20$ per share in exactly 3 months ($t_D = 0.25$ years). The continuously compounded risk-free rate is $r = 4.40\%$ per annum across all maturities. A 6-month ($T = 0.50$ years) OTC forward contract on this share is currently quoted at $F_{\text{mkt}} = \$84.50$. Determine the fair no-arbitrage forward price and demonstrate the exact cash-and-carry arbitrage profit on a 1,000-share transaction.

1. **Compute the present value of the known dividend I:**

$$I = 2.20\, e^{-0.044\times 0.25} = 2.20 \times 0.989060 = \$2.1759$$

2. **Compute the fair no-arbitrage forward price F^*:**

$$F^* = (S_0 - I)e^{rT} = (84.00 - 2.1759)\, e^{0.044\times 0.50} = 81.8241 \times 1.022244 = \$83.6441$$

3. **Identify mispricing and construct the arbitrage at $t = 0$:** Because *F_{text{mkt}} = $84.50 > F^* = $83.6441*, the forward contract is overpriced relative to the spot share. For 1,000 shares, the arbitrageur shorts forward contracts on 1,000 shares at $\$84.50$, buys 1,000 shares in the spot market for $\$84,000.00$, and finances the purchase with two risk-free loans:
   - Loan A: Borrow $\$2,175.93$ ($1,000 \times I$) for 3 months at 4.40% p.a.
   - Loan B: Borrow $\$81,824.07$ ($1,000 \times (S_0 - I)$) for 6 months at 4.40% p.a.
4. **Settlement at $t = 0.25$ years (3 months):** Receive dividends of $1,000 \times \$2.20 = \$2,200.00$. Repay Loan A in full: $\$2,175.93 \times e^{0.044\times 0.25} = \$2,200.00$. Net intermediate cash flow is $\$0.00$.
5. **Settlement at $t = 0.50$ years (6 months):** Deliver 1,000 shares against the short forward contract to receive $1,000 \times \$84.50 = \$84,500.00$. Repay Loan B in full: $\$81,824.07 \times e^{0.044\times 0.50} = \$83,644.14$.
6. **Riskless Arbitrage Profit at $T = 0.50$:**

$$\Pi_T = \$84,500.00 - \$83,644.14 = \$855.86$$

Notice that $\$855.86 = 1,000 \times (\$84.50 - \$83.6441)$, confirming the exact theoretical arbitrage spread.

Worked Example 3: Covered Interest Rate Parity and AUD/USD Forward Valuation

The spot exchange rate is $S_0 = 1.5340$ AUD per 1 USD. The continuously compounded 9-month risk-free interest rates are $r_{\text{AUD}} = 4.35\%$ p.a. and $r_{\text{USD}} = 4.85\%$ p.a. First, determine the fair 9-month forward rate F_0. Second, suppose an Australian importer holds an existing long forward contract to purchase USD 1,000,000 at a contracted delivery rate of $K = 1.5000$ AUD/USD with exactly 6 months ($\tau = 0.50$ years) remaining to maturity, and assume the same interest rates apply to the 6-month horizon. Calculate the current AUD mark-to-market value of this contract.

1. **Calculate the fair 9-month AUD/USD forward rate:**

$$F_0 = S_0 e^{(r_d - r_f)T} = 1.5340\, e^{(0.0435-0.0485)\times 0.75} = 1.5340\, e^{-0.00375} = 1.5340 \times 0.996257 = 1.5283 \text{ AUD/USD}$$

Because the AUD interest rate is lower than the USD interest rate, the USD trades at a forward discount (or equivalently, the AUD trades at a forward premium).

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2. **Value the existing 6-month long forward contract on USD 1,000,000 at $K = 1.5000$:**

$$f_t = \text{Notional}_{\text{USD}} \times (S_t e^{-r_f \tau} - K e^{-r_d \tau})$$

Computing each discounted leg per USD 1:

$$S_t e^{-r_f \tau} = 1.5340 \times e^{-0.0485 \times 0.50} = 1.5340 \times 0.976041 = 1.497247 \text{ AUD}$$

$$K e^{-r_d \tau} = 1.5000 \times e^{-0.0435 \times 0.50} = 1.5000 \times 0.978485 = 1.467727 \text{ AUD}$$

$$f_t = 1{,}000{,}000 \times (1.497247 - 1.467727) = \$29{,}520.68 \text{ AUD}$$

The existing long forward contract is an asset worth A$29,520.68 to the Australian importer.

1.4 Hedging Strategies Using Futures and Basis Risk

Corporate treasurers and fund managers routinely use futures contracts to lock in future purchase or sale prices. A **short hedge** involves taking a short futures position to protect the value of an asset already owned or expected to be produced and sold in the future (e.g., an Australian grain exporter hedging a forthcoming wheat harvest). A **long hedge** involves taking a long futures position to fix the purchase price of an asset required in the future (e.g., an Australian airline locking in future aviation fuel costs or a fund manager locking in equity entry prices ahead of anticipated superannuation inflows).

Basis Risk and Effective Hedged Price

In real-world hedging, the asset being hedged rarely matches the futures underlying in exact grade, location, or expiry date. The difference between the prevailing spot price of the hedged asset and the futures price of the contract used is defined as the **basis**:

$$b_t = S_t - F_t$$

Where: b_t is the basis at time t; S_t is the spot price of the asset being hedged; and F_t is the futures price of the contract selected.

Suppose a hedger initiates a short hedge at t_1 at futures price F_1 and closes both the physical sale and the futures position at t_2 when spot and futures prices are S_2 and F_2. The total effective price realised per unit is the spot sale proceeds plus the futures gain or loss:

$$S_{\text{eff, short}} = S_2 + (F_1 - F_2) = F_1 + (S_2 - F_2) = F_1 + b_2$$

Similarly, a long hedger buying the asset at t_2 pays an effective price of:

$$S_{\text{eff, long}} = S_2 - (F_2 - F_1) = F_1 + (S_2 - F_2) = F_1 + b_2$$

Where: F_1 is the initial futures entry price; F_2 is the closing futures price at hedge lift; S_2 is the spot price at t_2; and $b_2 = S_2 - F_2$ is the closing basis at t_2.

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Notice that the hedger eliminates exposure to absolute spot price movements $S_2 - S_1$ and replaces it entirely with uncertainty over the closing basis b_2. An unexpected increase in the basis ($b_2 > b_1$) is called a **strengthening of the basis**, which benefits a short hedger (higher effective selling price) and harms a long hedger (higher effective purchase cost). Conversely, a decline in the basis ($b_2 < b_1$) is a **weakening of the basis**, benefiting a long hedger and harming a short hedger. To minimise basis risk, hedgers select the most liquid futures contract whose expiry date occurs just *after* the planned physical transaction date, avoiding delivery-month volatility and physical delivery obligations.

Cross-Hedging and the Minimum-Variance Hedge Ratio (MVHR)

When no futures contract exists on the exact asset to be hedged—such as jet kerosene hedged with crude oil or gasoil futures, or a corporate bond portfolio hedged with 10-Year Commonwealth Treasury Bond futures—the hedger executes a **cross-hedge**. Because the price changes of the spot asset (ΔS) and the futures contract (ΔF) do not move one-for-one, hedging unit-for-unit is generally suboptimal.

Let h denote the hedge ratio (size of the futures position relative to the spot exposure). For a short hedger, the change in total hedged portfolio value over the hedge horizon is $\Delta V = \Delta S - h\Delta F$. Minimising the variance of ΔV:

$$\text{Var}(\Delta V) = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

Differentiating with respect to h and setting the first derivative equal to zero gives the **Minimum-Variance Hedge Ratio (MVHR)** h^* and the corresponding **hedge effectiveness** R^2:

$$h^* = \rho\frac{\sigma_S}{\sigma_F} = \frac{\text{Cov}(\Delta S, \Delta F)}{\text{Var}(\Delta F)}, \qquad R^2 = \rho^2 = \frac{(h^*)^2\sigma_F^2}{\sigma_S^2}$$

Where: σ_S is the standard deviation of changes in the spot price ΔS over the hedge horizon; σ_F is the standard deviation of changes in the futures price ΔF; ρ is the correlation coefficient between ΔS and ΔF; and R^2 is the proportion of spot price variance eliminated by the optimal hedge.

In empirical regression analysis, h^* equals the OLS slope coefficient from regressing ΔS on ΔF. Given a physical exposure quantity Q_A and a single futures contract size Q_F, the optimal number of futures contracts N^* is:

$$N^* = h^*\frac{Q_A}{Q_F}$$

When hedging over longer horizons with daily marked-to-market futures, a dollar change in futures generates immediate variation margin cash flows, whereas spot value changes accrue only at maturity. Multiplying N^* by e^{-rT}—or equivalently using the dollar-value ratio $N^*_{\text{tailed}} = h^*\frac{V_A}{V_F}$, where $V_A = Q_A S_0$ is the dollar value of the spot position and $V_F = Q_F F_0$ is the dollar notional value of one futures contract—performs **tailing the hedge** to eliminate interest-rate timing mismatches on variation margin.

In empirical implementation, hedgers must carefully select the estimation window and return horizon used to compute σ_S, σ_F, and ρ. Matching the regression interval to the planned hedge holding period reduces noise from non-synchronous closing prices, while rolling estimation windows prevent stale historical correlations from distorting h^* during macroeconomic regime shifts. A breakdown of hedging error sources is summarised below.

Source of Hedge Imperfection	Economic Cause	Mitigation Technique
Asset Mismatch (Cross-Hedge)	Physical asset differs in grade, quality, or location from futures specification.	Apply OLS minimum-variance hedge ratio $h^* = \rho(\sigma_S/\sigma_F)$.

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Source of Hedge Imperfection	Economic Cause	Mitigation Technique
Maturity Mismatch	Spot sale/purchase date falls prior to futures contract expiration.	Select nearest liquid contract expiring just after hedge horizon.
Contract Granularity	Optimal contract count N^* is fractional and must be rounded to integer lots.	Round to nearest integer; residual exposure accepted as minor tracking error.
Variation Margin Timing	Daily cash margining on futures vs terminal settlement on spot position.	Apply tailed hedge adjustment $N^*_{\text{tailed}} = h^*(V_A/V_F)$.

Worked Example 4: Cross-Hedging Jet Fuel Exposure for an Australian Airline

An Australian domestic airline anticipates purchasing $2,500,000$ gallons of aviation jet fuel in three months. Because no liquid AUD jet fuel futures contract exists, the treasury team cross-hedges using heating oil futures, where each contract covers $42,000$ gallons. Historical three-month price changes yield a spot jet fuel standard deviation of $\sigma_S = 0.038$ AUD/gallon, a futures price standard deviation of $\sigma_F = 0.042$ AUD/gallon, and a correlation of $\rho = 0.92$. Compute the MVHR, hedge effectiveness, and optimal number of contracts. Also calculate the net effective fuel cost if spot fuel rises from $\$3.10$ to $\$3.35$/gal while futures rise from $\$3.02$ to $\$3.29$/gal.

1. **Calculate the Minimum-Variance Hedge Ratio h^*:**

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.92 \times \frac{0.038}{0.042} = 0.8324$$

2. **Calculate Hedge Effectiveness R^2:**

$$R^2 = \rho^2 = (0.92)^2 = 0.8464 \text{ (84.64\% of price variance eliminated)}$$

3. **Determine the Optimal Number of Long Futures Contracts N^*:**

$$N^* = h^* \frac{Q_A}{Q_F} = 0.8324 \times \frac{2,500,000}{42,000} = 49.55 \approx 50 \text{ contracts (long)}$$

4. **Evaluate the Realised Effective Cost with 50 Contracts:**
   - Unhedged spot purchase cost at t_2: $2,500,000 \times \$3.35 = \$8,375,000$.
   - Futures gain on 50 long contracts: $50 \times 42,000 \times (\$3.29 - \$3.02) = 2,100,000 \times \$0.27 = \$567,000$.
   - Net hedged fuel expenditure: $\$8,375,000 - \$567,000 = \$7,808,000$, or $\$3.1232$/gallon (compared with $\$3.35$/gallon unhedged).

Equity Portfolio Beta Hedging with ASX SPI 200 Futures

Institutional equity managers frequently wish to alter the systematic market risk (beta, β) of an Australian share portfolio without liquidating physical holdings and incurring brokerage or capital gains tax events. Suppose a portfolio with market value V_P has a CAPM beta of β relative to the S&P/ASX 200 index. Let $V_F = F_0 \times \text{Multiplier}$ be the notional AUD value of one SPI 200 futures contract. To shift the portfolio's effective beta from β to a target beta β^* (where $\beta^* = 0$ achieves complete market neutrality), the required number of futures contracts N^* is:

$$N^* = (\beta^* - \beta)\frac{V_P}{V_F}$$

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Where: β^* is the target portfolio beta; β is the current portfolio beta; V_P is the current market value of the equity portfolio; and V_F is the current dollar notional value of one index futures contract. A negative value of N^* indicates a **short** futures position, whereas a positive value indicates a **long** futures position.

Rolling the Hedge Forward and Liquidity Risk

When a hedging horizon exceeds the maturity of the most liquid nearby futures contracts, a hedger employs a **stack-and-roll strategy**: opening the full futures position in a near-term contract and, just prior to expiry, closing it out and simultaneously opening a new position in the subsequent contract month. While stack-and-roll lowers transaction costs and tracks near-term price changes closely, each roll introduces **rollover basis risk** (exposure to the calendar spread between expiring and deferred contracts) and acute **funding liquidity risk**. The collapse of Metallgesellschaft Refining and Marketing in 1993 demonstrated that even a sound long-term hedge can fail if short-term futures losses trigger massive daily variation margin calls before long-term physical delivery profits are realised.

Worked Example 5: Portfolio Beta Adjustment Using ASX SPI 200 Futures

An Australian superannuation fund manages a diversified domestic equity portfolio valued at $V_P = \$48,000,000$ with an estimated beta of $\beta = 1.25$ relative to the S&P/ASX 200. Anticipating short-term macroeconomic volatility ahead of an RBA policy decision, the investment committee resolves to reduce the portfolio's beta to $\beta^* = 0.75$ over the next three months ($T = 0.25$ years). Currently, the S&P/ASX 200 spot index stands at $S_0 = 7,780$, the 3-month SPI 200 futures contract trades at $F_0 = 7,800$ (contract multiplier A$25), the risk-free rate is $r = 4.20\%$ p.a., and the index dividend yield is $q = 3.17\%$ p.a. (continuous). Calculate the required futures trade and verify the hedged portfolio return if the S&P/ASX 200 spot index drops by 10% over three months to $S_T = 7,002$.

1. **Compute the notional value of one SPI 200 futures contract V_F:**

$$V_F = 7,800 \times \$25 = \$195,000$$

2. **Compute the required number of futures contracts N^*:**

$$N^* = (\beta^* - \beta)\frac{V_P}{V_F} = (0.75 - 1.25) \times \frac{\$48,000,000}{\$195,000} = -0.50 \times 246.1538 = -123.08 \approx -123 \text{ contracts}$$

   The fund must **short 123 SPI 200 futures contracts**.

3. **Verify performance under a 10% index decline over 3 months ($S_T = 7,002$):**
   - Index capital return = -10.00%; 3-month dividend return $\approx 3.17\% \times 0.25 = +0.79\%$; total index return $R_m = -9.21\%$.
   - Risk-free return for 3 months: $R_f = 4.20\% \times 0.25 = +1.05\%$.
   - Expected physical portfolio return via CAPM:

$$R_P = R_f + \beta(R_m - R_f) = 1.05\% + 1.25(-9.21\% - 1.05\%) = 1.05\% - 12.825\% = -11.775\%$$

     Physical portfolio loss: $\$48,000,000 \times (-0.11775) = -\$5,652,000$, leaving physical value at $\$42,348,000$.
   - At expiry $T = 0.25$, the futures price converges to $F_T = S_T = 7,002$. Gain on 123 short SPI 200 contracts:

$$\text{Futures PnL} = 123 \times \$25 \times (7,800 - 7,002) = 3,075 \times 798 = +\$2,453,850$$

   - Total hedged portfolio value = $\$42,348,000 + \$2,453,850 = \$44,801,850$, representing a net return of -6.663%.
   - Check against theoretical target beta $\beta^* = 0.75$:

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$$R_P^* = 1.05\% + 0.75(-9.21\% - 1.05\%) = 1.05\% - 7.695\% = -6.645\%$$

The 0.018% difference reflects rounding 123.08 contracts to the nearest integer 123, confirming the precision of beta-hedging mechanics.

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