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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Corporate Finance

Course Code: BFC2140 • Module 3 Notes

Module 3: Risk, Return, Portfolio Theory & the Capital Asset Pricing Model (CAPM)

3.1 Measuring Historical and Expected Return and Risk

Every capital allocation and security valuation decision rests on the trade-off between expected return and risk. When investors purchase shares or corporate bonds on the Australian Securities Exchange (ASX), their total return comprises two components: an **income yield** (cash dividends or coupons received during the holding period) and a **capital gain** (or loss) arising from changes in market price.

Holding Period Return, Arithmetic Mean, and Geometric Mean

For a single holding period from $t-1$ to t, the total percentage **Holding Period Return (HPR)** on an ordinary share is:

$$R_t = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where: R_t is the total return in period t; D_t is the cash dividend paid during period t; P_{t-1} is the beginning share price; and P_t is the ending share price.

Across a multi-period historical sample of T periods $(R_1, R_2, \dots, R_T)$, analysts summarise average performance using two distinct averages:

- **Arithmetic Mean Return ($\bar{R}_A$):** The simple average of single-period returns, representing an unbiased estimate of the expected return over a single future period drawn from the same stationary distribution:

$$\bar{R}_A = \frac{1}{T}\sum_{t=1}^{T} R_t$$

- **Geometric Mean Return ($\bar{R}_G$, or Compound Annual Growth Rate):** The constant compound rate per period that links initial wealth to terminal wealth over the multi-year sample:

$$\bar{R}_G = \left[\prod_{t=1}^{T}(1 + R_t)\right]^{\frac{1}{T}} - 1$$

Where: T is the number of periods; and R_t is the realised return in period t. Because of volatility drag (Jensen's inequality), whenever returns vary over time, $\bar{R}_G < \bar{R}_A$, with the approximate difference equal to half the return variance: $\bar{R}_G \approx \bar{R}_A - \frac{1}{2}\sigma^2$.

Ex-Ante Expected Return, Variance, and Standard Deviation

In corporate finance, investment decisions depend on forward-looking (**ex-ante**) probability distributions across S possible future economic states $(s = 1, \dots, S)$, each occurring with probability p_s such that $\sum_{s=1}^{S} p_s = 1$. The ex-ante **expected return** $E(R)$, **variance** σ^2, and **standard deviation** σ (total volatility) are defined as:

$$E(R) = \sum_{s=1}^{S} p_s R_s, \qquad \sigma^2 = \sum_{s=1}^{S} p_s \left[R_s - E(R)\right]^2, \qquad \sigma = \sqrt{\sigma^2}$$

Where: p_s is the probability of economic state s; R_s is the asset return in state s; $E(R)$ is the probability-weighted mean return; σ^2 is the variance of returns; and σ is the standard deviation in percentage terms.

When estimating volatility from historical (**ex-post**) return data over T observations, the sample variance divides squared deviations by $T-1$ (Bessel's correction) to obtain an unbiased estimator of population variance:

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$$s^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R}_A)^2, \qquad s = \sqrt{s^2}$$

Where: s^2 is the sample variance; s is the sample standard deviation; and $\bar{R}_A$ is the sample arithmetic mean.

Historical Evidence on Australian Capital Markets

Long-horizon empirical studies of Australian financial markets—spanning the S&P/ASX 200 Accumulation Index, Commonwealth Government Bonds, and 90-day Bank Accepted Bills—confirm a persistent positive relationship between average return and return volatility across asset classes. Equities deliver the highest long-term nominal return (historically averaging roughly 10%–11% p.a. including dividends) alongside the highest annual standard deviation (16%–20% p.a.), followed by long-term ACGBs, corporate investment-grade debt, and money-market bills. The historical spread between Australian equity returns and the risk-free rate forms an important empirical benchmark for estimating the **Market Risk Premium (MRP)**.

Worked Example 1: Historical Return Statistics and Ex-Ante Scenario Risk

Part A — Ex-Post Historical Sample: An ASX-listed energy infrastructure company delivered annual total returns of $+14.0\%$, -6.0%, $+18.0\%$, and $+10.0\%$ over four consecutive years.

1. **Arithmetic Mean:**

$$\bar{R}_A = \frac{0.14 - 0.06 + 0.18 + 0.10}{4} = \frac{0.36}{4} = 0.0900 = 9.00\%\text{ p.a.}$$

2. **Geometric Mean (CAGR):**

$$\bar{R}_G = [(1.14)(0.94)(1.18)(1.10)]^{1/4} - 1 = (1.390903)^{0.25} - 1 = 1.08599 - 1 = 8.60\%\text{ p.a.}$$

3. **Sample Variance and Standard Deviation ($T-1=3$):**

$$s^2 = \frac{(0.14-0.09)^2 + (-0.06-0.09)^2 + (0.18-0.09)^2 + (0.10-0.09)^2}{4-1}$$

$$s^2 = \frac{0.0025 + 0.0225 + 0.0081 + 0.0001}{3} = \frac{0.0332}{3} = 0.011067, \qquad s = \sqrt{0.011067} = 10.52\%$$

Part B — Ex-Ante Probability Distribution: Analysts forecast next year's returns across three Australian macroeconomic states for Stock X (Resources) and Stock Y (Consumer Staples):

Economic State (s)	Probability (p_s)	Stock X Return ($R_{X,s}$)	Stock Y Return ($R_{Y,s}$)
Boom	0.30	26.0%	10.0%
Normal Growth	0.50	11.0%	8.0%
Recession	0.20	-14.0%	5.0%

- $E(R_X) = 0.30(26\%) + 0.50(11\%) + 0.20(-14\%) = 10.50\%$

$$\sigma_X^2 = 0.30(0.26-0.105)^2 + 0.50(0.11-0.105)^2 + 0.20(-0.14-0.105)^2 = 0.019225 \implies \sigma_X = 13.87\%$$

- $E(R_Y) = 0.30(10\%) + 0.50(8\%) + 0.20(5\%) = 8.00\%$

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$$\sigma_Y^2 = 0.30(0.10 - 0.08)^2 + 0.50(0.08 - 0.08)^2 + 0.20(0.05 - 0.08)^2 = 0.000300 \implies \sigma_Y = 1.73\%$$

3.2 Covariance, Correlation, and Two-Asset Portfolio Theory

Rational investors evaluate securities not in isolation, but as components of an overall portfolio. While a portfolio's expected return is simply the weighted linear average of its constituent assets' expected returns, portfolio risk depends critically on how asset returns move together across states of nature.

Covariance and Correlation Coefficient

Covariance measures the directional linear co-movement between two risky assets A and B. Ex-ante covariance across S states is:

$$\text{Cov}(R_A, R_B) = \sigma_{AB} = \sum_{s=1}^{S} p_s \left[R_{A,s} - E(R_A)\right] \left[R_{B,s} - E(R_B)\right]$$

Where: p_s is the probability of state s; $R_{A,s}, R_{B,s}$ are state returns; and $E(R_A), E(R_B)$ are expected returns. (For historical data of length T, sample covariance replaces p_s with $\frac{1}{T-1}\sum_{t=1}^{T}$.) Because covariance depends on return units and scales, analysts standardise it into the dimensionless **correlation coefficient** ρ_{AB} bounded strictly on $[-1, +1]$:

$$\rho_{AB} = \frac{\text{Cov}(R_A, R_B)}{\sigma_A \sigma_B} \iff \text{Cov}(R_A, R_B) = \rho_{AB}\sigma_A\sigma_B$$

Where: $\rho_{AB} = +1$ denotes perfect positive linear co-movement; $\rho_{AB} = 0$ denotes zero linear correlation; and $\rho_{AB} = -1$ denotes perfect negative linear co-movement.

Two-Asset Portfolio Expected Return, Variance, and Minimum Variance

Consider a portfolio allocating weight w_A to Asset A and weight $w_B = 1 - w_A$ to Asset B. Portfolio expected return $E(R_p)$ and portfolio variance σ_p^2 are given by Markowitz's two-asset equations:

$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \text{Cov}(R_A, R_B) = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{AB}\sigma_A\sigma_B$$

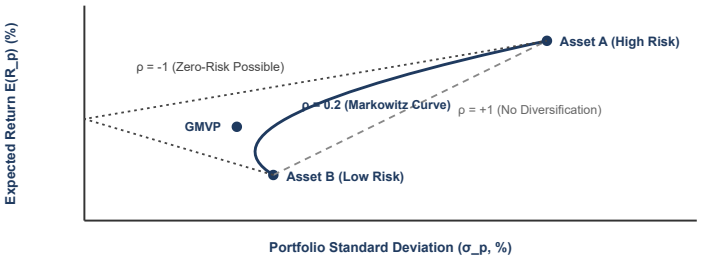
Where: w_A, w_B are portfolio value weights summing to 1; σ_A, σ_B are individual standard deviations; and $\sigma_p = \sqrt{\sigma_p^2}$ is portfolio standard deviation.

The fundamental insight of Harry Markowitz (1952) is that whenever $\rho_{AB} < +1$, portfolio standard deviation σ_p is strictly *less* than the weighted average of individual standard deviations ($w_A\sigma_A + w_B\sigma_B$). Thus, diversification reduces total risk without necessarily sacrificing expected return. Differentiating σ_p^2 with respect to w_A and setting the derivative to zero yields the weights of the **Global Minimum Variance Portfolio (GMVP)**:

$$w_A^{\min} = \frac{\sigma_B^2 - \text{Cov}(R_A, R_B)}{\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(R_A, R_B)}, \qquad w_B^{\min} = 1 - w_A^{\min}$$

Where: $w_A^{\min}$ and $w_B^{\min}$ are the asset weights minimising total portfolio variance.

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Worked Example 2: Two-Stock ASX Portfolio and Minimum Variance Weights

An Australian equity manager allocates capital between two ASX-listed blue-chip companies: BHP Group ($E(R_A) = 13.0\%$, $\sigma_A = 22.0\%$) and Woolworths Group ($E(R_B) = 8.0\%$, $\sigma_B = 12.0\%$). The correlation between their returns is $\rho_{AB} = 0.20$.

1. **Compute Covariance between BHP and Woolworths:**

$$\text{Cov}(R_A, R_B) = \rho_{AB}\sigma_A\sigma_B = 0.20 \times 0.22 \times 0.12 = 0.00528$$

2. **Evaluate an equal-weight ($w_A = 0.50, w_B = 0.50$) portfolio:**

$$E(R_p) = 0.50(13.0\%) + 0.50(8.0\%) = 10.50\%$$

$$\sigma_p^2 = (0.50)^2(0.22)^2 + (0.50)^2(0.12)^2 + 2(0.50)(0.50)(0.00528)$$

$$\sigma_p^2 = 0.01210 + 0.00360 + 0.00264 = 0.01834 \implies \sigma_p = \sqrt{0.01834} = 13.54\%$$

Note that the weighted average of standalone volatilities is 0.50(22%) + 0.50(12%) = 17.00%; diversification trims 3.46 percentage points of volatility.

3. **Calculate the Global Minimum Variance Portfolio (GMVP):**

$$w_A^{\min} = \frac{(0.12)^2 - 0.00528}{(0.22)^2 + (0.12)^2 - 2(0.00528)} = \frac{0.00912}{0.05224} = 0.1746 \ (17.46\%)$$

$$w_B^{\min} = 1 - 0.1746 = 0.8254 \ (82.54\%)$$

$$E(R_{\text{GMVP}}) = 0.1746(13.0\%) + 0.8254(8.0\%) = 8.87\%$$

$$\sigma_{\text{GMVP}}^2 = (0.1746)^2(0.0484) + (0.8254)^2(0.0144) + 2(0.1746)(0.8254)(0.00528) = 0.012808 \implies \sigma_{\text{GMVP}} = 11.32\%$$

Remarkably, adding 17.46% of high-volatility BHP to 82.54% Woolworths reduces total volatility from 12.00% down to 11.32% while raising expected return from 8.00% to 8.87%.

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3.3 Multi-Asset Diversification, Efficient Frontier & Capital Allocation

Generalising Markowitz's framework to an N-asset universe reveals why diversification works so powerfully in broad equity markets. The variance of an N-asset portfolio with weights $w_1, \ldots, w_N$ is:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} w_i w_j \mathrm{Cov}(R_i, R_j)$$

Where: there are N individual variance terms and $N(N-1)$ pairwise covariance terms. For an equally weighted portfolio ($w_i = 1/N$) with average variance $\bar{\sigma}^2$ and average pairwise covariance $\overline{\mathrm{Cov}}$, portfolio variance simplifies to:

$$\sigma_p^2 = \frac{1}{N}\bar{\sigma}^2 + \left(1 - \frac{1}{N}\right)\overline{\mathrm{Cov}}$$

As N becomes large (for example, holding a broad ASX 200 index portfolio), $\frac{1}{N}\bar{\sigma}^2 \to 0$ and $\sigma_p^2 \to \overline{\mathrm{Cov}}$. This mathematical limit partitions total risk into two distinct components:

- **Unsystematic (Idiosyncratic / Firm-Specific) Risk:** Risk unique to individual firms or industries (e.g., a factory outage, clinical trial failure, or management dispute) that is eliminated through broad portfolio diversification.
- **Systematic (Market / Non-Diversifiable) Risk:** Economy-wide macroeconomic risk (RBA cash rate surprises, GDP downturns, commodity price shocks, inflation regimes) captured by average covariance $\overline{\mathrm{Cov}}$, which cannot be diversified away regardless of how many equities are held.

To illustrate how rapidly idiosyncratic risk dissipates on the ASX, consider an equal-weighted portfolio drawn from stocks with typical annualised variance $\bar{\sigma}^2 = 0.0900$ ($\bar{\sigma} = 30.0\%$) and average pairwise covariance $\overline{\mathrm{Cov}} = 0.0225$ (corresponding to an average correlation $\bar{\rho} = 0.25$ and systematic standard deviation $\sqrt{0.0225} = 15.0\%$):

Number of Stocks (N)	Idiosyncratic Share $\frac{1}{N}(\bar{\sigma}^2 - \overline{\mathrm{Cov}})$	Total Portfolio Variance (σ_p^2)	Portfolio Volatility (σ_p)	Systematic Variance Share (%)
1	0.06750	0.09000	30.00%	25.0%
2	0.03375	0.05625	23.72%	40.0%
5	0.01350	0.03600	18.97%	62.5%
10	0.00675	0.02925	17.10%	76.9%
25	0.00270	0.02520	15.87%	89.3%
50	0.00135	0.02385	15.44%	94.3%
200 (ASX 200 Proxy)	0.00034	0.02284	15.11%	98.5%

Holding just 25 diversified stocks across distinct Australian industry sectors eliminates 96% of diversifiable variance, leaving systematic market risk responsible for nearly 90% of portfolio variance.

Efficient Frontier, Capital Allocation Line (CAL), and Capital Market Line (CML)

Across all N risky assets, the upper boundary of the feasible set lying above the Global Minimum Variance Portfolio is the **Markowitz Efficient Frontier**: the locus of portfolios offering the highest expected return for each level of total volatility σ_p. Any portfolio inside or below this curve is mean-variance dominated.

When investors can also borrow or lend at a domestic risk-free interest rate R_f (proxied in Australia by 90-day Bank Accepted Bills or short-dated ACGBs), combining R_f with any risky portfolio P generates a linear **Capital Allocation Line (CAL)** in $(E(R), \sigma)$ space. The slope of the CAL is portfolio P's **Sharpe Ratio** (reward-to-variability ratio):

$$E(R_C) = R_f + \left[\frac{E(R_P) - R_f}{\sigma_P}\right]\sigma_C, \qquad \text{Sharpe Ratio}_P = \frac{E(R_P) - R_f}{\sigma_P}$$

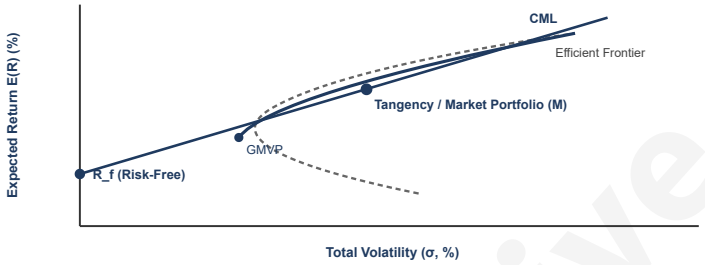
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Where: $E(R_C)$ and σ_C are the expected return and standard deviation of the combined portfolio (investing fraction y in P and $(1-y)$ in R_f); and $\frac{E(R_P)-R_f}{\sigma_P}$ is the excess return per unit of total volatility.

To maximise expected return for every level of risk, investors rotate the CAL upward until it is tangent to the efficient frontier at a unique risky portfolio T—the **Tangency Portfolio** (which maximises the Sharpe Ratio). Under homogeneous investor expectations, every investor holds the same risky tangency portfolio—which in market equilibrium must equal the value-weighted **Market Portfolio (M)**—combined with risk-free lending or borrowing depending on personal risk aversion (the **Two-Fund Separation Theorem**). That optimal CAL through M is the **Capital Market Line (CML)**:

$$E(R_C) = R_f + [\frac{E(R_M) - R_f}{\sigma_M}]\sigma_C$$

Where: $E(R_M)$ and σ_M are the expected return and standard deviation of the market portfolio.



Worked Example 3: Sharpe Ratio Comparison and Capital Allocation Target

Suppose the Australian risk-free rate is $R_f = 4.0\%$ p.a. An investor compares two candidate risky portfolios: Portfolio Alpha ($E(R_\alpha) = 10.6\%, \sigma_\alpha = 15.0\%$) and Portfolio Beta ($E(R_\beta) = 12.1\%, \sigma_\beta = 18.0\%$).

1. **Identify the superior risky portfolio via Sharpe Ratio:**

$$\text{Sharpe}_\alpha = \frac{10.6\% - 4.0\%}{15.0\%} = \frac{6.6\%}{15.0\%} = 0.440$$

$$\text{Sharpe}_\beta = \frac{12.1\% - 4.0\%}{18.0\%} = \frac{8.1\%}{18.0\%} = 0.450$$

Portfolio Beta offers a higher excess return per unit of total volatility (0.450 > 0.440) and therefore lies on a steeper CAL.

2. **Construct a combined portfolio with a target volatility of $\sigma_C = 12.0\%$:**

$$\sigma_C = y \cdot \sigma_\beta \implies y = \frac{12.0\%}{18.0\%} = 0.6667 \text{ (66.67\% in Portfolio Beta, 33.33\% in } R_f)$$

$$E(R_C) = 4.0\% + 0.6667(12.1\% - 4.0\%) = 4.0\% + 5.40\% = 9.40\% \text{ p.a.}$$

3.4 The Capital Asset Pricing Model (CAPM) and Beta Estimation

While the CML prices only well-diversified efficient portfolios using total volatility σ, corporate managers and investors must value *individual* stocks and capital projects whose idiosyncratic risk is diversified away inside investors' portfolios. Developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), the **Capital Asset Pricing Model (CAPM)** proves that in competitive market equilibrium, only **systematic risk** commands a risk premium.

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Systematic Risk Beta and the Security Market Line (SML)

An individual asset i's contribution to the variance of the market portfolio M is governed by its covariance with the market return, $\text{Cov}(R_i, R_m)$. Normalising this covariance by the market portfolio's variance yields **Beta (β_i)**:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} = \rho_{i,m}\left(\frac{\sigma_i}{\sigma_m}\right)$$

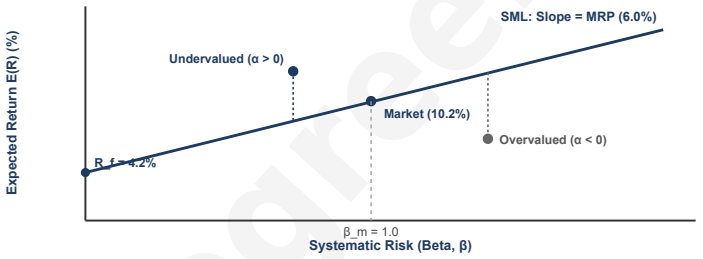
Where: β_i measures the sensitivity of asset i's return to market-wide movements; $\text{Cov}(R_i, R_m)$ is the covariance between asset i and the market portfolio; σ_m^2 is the variance of the market portfolio; and $\rho_{i,m}$ is the correlation between asset i and the market.

By definition, the market portfolio has $\beta_m = 1.0$ and the risk-free asset has $\beta_f = 0$. Aggressive cyclical equities (e.g., Australian mining and discretionary retail firms) typically exhibit $\beta > 1$, whereas defensive equities (e.g., supermarkets, regulated utilities, and telecommunications) typically exhibit $0 < \beta < 1$. Under the CAPM, the equilibrium expected return on any individual asset or portfolio i is given by the **Security Market Line (SML)**:

$$E(R_i) = R_f + \beta_i\left[E(R_m) - R_f\right]$$

Where: $E(R_i)$ is the CAPM required return (cost of equity); R_f is the risk-free rate; $E(R_m) - R_f$ is the Market Risk Premium (MRP); and β_i is the asset's systematic risk.

If an analyst forecasts an expected return $\hat{E}(R_i)$ (from fundamental valuation or DDM) that differs from the CAPM equilibrium benchmark, the discrepancy is **Jensen's alpha**: $\alpha_i = \hat{E}(R_i) - E(R_i)_{\text{CAPM}}$. Securities plotting *above* the SML ($\alpha_i > 0$) offer expected returns exceeding fair compensation for their systematic risk and are **undervalued**; securities plotting *below* the SML ($\alpha_i < 0$) are **overvalued**.



Feature	Capital Market Line (CML)	Security Market Line (SML)
Horizontal Axis Risk Measure	Total Volatility (σ_p)	Systematic Risk (β_i)
Applicable Investments	Efficient portfolios only (combinations of R_f and M)	All individual assets, projects, and portfolios (efficient or inefficient)
Slope of Line	Market Sharpe Ratio: $\frac{E(R_m)-R_f}{\sigma_m}$	Market Risk Premium: $E(R_m) - R_f$
Portfolio Additivity	Non-linear for inefficient assets	Strictly linear: $\beta_p = \sum_{i=1}^{N} w_i\beta_i$

Empirical Beta Estimation on the ASX and Leverage Adjustments

In Australian practice, analysts estimate equity beta by running an Ordinary Least Squares (OLS) **market model regression** using historical monthly excess returns over a 4- to 5-year window against the S&P/ASX 200 Accumulation Index:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

Where: the estimated slope $\hat{\beta}_i$ measures historical systematic risk; R^2 measures the proportion of return variance attributable to systematic market factors ($(1 - R^2)$ represents diversifiable firm-specific risk); and $\varepsilon_{i,t}$ is the zero-mean idiosyncratic residual.

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Because historical OLS betas exhibit mean reversion toward 1.0 over time, practitioners frequently apply the **Blume adjustment**: $\beta_{\text{adjusted}} = \frac{2}{3}\hat{\beta}_{\text{OLS}} + \frac{1}{3}(1.0)$. Moreover, equity beta (β_E) reflects both underlying business operating risk (**asset beta**, β_A) and financial leverage risk. Assuming risk-free debt ($\beta_D = 0$) and a constant debt-to-equity ratio with corporate tax rate t_c (the Hamada relationship), asset and equity betas link via:

$$\beta_A = \frac{\beta_E}{1 + (1 - t_c)\frac{D}{E}}, \qquad \beta_E = \beta_A\left[1 + (1 - t_c)\frac{D}{E}\right]$$

Where: β_A is the unlevered asset beta; β_E is the levered equity beta; D/E is the market-value debt-to-equity ratio; and t_c is the corporate tax rate.

Worked Example 4: Beta Calculation, SML Alpha, and Unlevering/Relevering Beta

An analyst collects ASX market data where the risk-free rate (10-year ACGB yield) is $R_f = 4.20\%$ and the expected return on the S&P/ASX 200 is $E(R_m) = 10.20\%$ (giving an MRP of 6.00%) with market standard deviation $\sigma_m = 15.0\%$. Stock K has total volatility $\sigma_K = 24.0\%$ and correlation $\rho_{K,m} = 0.75$ with the market. Stock K currently trades at $P_0 = \$20.00$, is expected to pay $D_1 = \$1.10$ next year, and has expected perpetual dividend growth of 6.0%.

1. **Compute Stock K's Beta and CAPM Required Return:**

$$\beta_K = \rho_{K,m}\left(\frac{\sigma_K}{\sigma_m}\right) = 0.75 \times \left(\frac{0.24}{0.15}\right) = 0.75 \times 1.60 = 1.20$$

$$E(R_K)_{\text{CAPM}} = 4.20\% + 1.20(10.20\% - 4.20\%) = 4.20\% + 7.20\% = 11.40\% \text{ p.a.}$$

2. **Test for Mispricing via Jensen's Alpha:** Based on market price and DDM fundamentals, Stock K's expected return is:

$$\hat{E}(R_K) = \frac{D_1}{P_0} + g = \frac{\$1.10}{\$20.00} + 0.060 = 5.50\% + 6.00\% = 11.50\%$$

$$\alpha_K = 11.50\% - 11.40\% = +0.10\% \text{ (or } +10 \text{ bps)}$$

   Because $\alpha_K > 0$, Stock K plots slightly above the SML and is marginally **undervalued** (its intrinsic DDM price at $r_E = 11.40\%$ is $\frac{1.10}{0.114 - 0.060} = \$20.37$ versus \$20.00 market price).

3. **Unlever and Relever Beta for a Comparable Firm:** Suppose Stock K operates with a market debt-to-equity ratio of $D/E = 0.50$ under Australia's 30% corporate tax rate ($t_c = 0.30$). A private division in the same industry will operate at $D/E = 0.80$.

$$\beta_A = \frac{1.20}{1 + (1 - 0.30)(0.50)} = \frac{1.20}{1.35} = 0.8889$$

$$\beta_{E,\text{new}} = 0.8889\,[1 + (1 - 0.30)(0.80)] = 0.8889 \times 1.56 = 1.3867$$

$$r_{E,\text{new}} = 4.20\% + 1.3867(6.00\%) = 12.52\%$$

Worked Example 5: Empirical OLS Beta Estimation From Historical ASX Returns

To illustrate how beta is estimated directly from historical market data, suppose an ASX-listed industrial firm (Pacific Freight Ltd) and the S&P/ASX 200 Accumulation Index recorded the following annual excess returns ($r_t = R_t - R_{f,t}$) over 5 years:

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Year	Excess Return $r_{P,t}$ (%)	Market Excess $r_{m,t}$ (%)	$r_{P,t} - \bar{r}_P$ (%)	$r_{m,t} - \bar{r}_m$ (%)	Cross-Product $(r_{P,t} - \bar{r}_P)(r_{m,t} - \bar{r}_m)$	Squared Market $(r_{m,t} - \bar{r}_m)^2$
1	14.0%	10.0%	+6.0%	+4.0%	0.00240	0.00160
2	-8.0%	-6.0%	-16.0%	-12.0%	0.01920	0.01440
3	20.0%	14.0%	+12.0%	+8.0%	0.00960	0.00640
4	2.0%	4.0%	-6.0%	-2.0%	0.00120	0.00040
5	12.0%	8.0%	+4.0%	+2.0%	0.00080	0.00040
Mean / Sum	$\bar{r}_P = 8.0\%$	$\bar{r}_m = 6.0\%$	**Sum = 0.0%**	**Sum = 0.0%**	**Sum = 0.03320**	**Sum = 0.02320**

1. **Compute Sample Covariance and Sample Market Variance ($T - 1 = 4$):**

$$\widehat{\mathrm{Cov}}(r_P, r_m) = \frac{0.03320}{5-1} = 0.00830, \qquad \hat{\sigma}_m^2 = \frac{0.02320}{5-1} = 0.00580$$

2. **Estimate OLS Beta, Regression Alpha, and Blume-Adjusted Beta:**

$$\hat{\beta}_{\text{OLS}} = \frac{\widehat{\mathrm{Cov}}(r_P, r_m)}{\hat{\sigma}_m^2} = \frac{0.00830}{0.00580} = 1.4310$$

$$\hat{\alpha}_{\text{OLS}} = \bar{r}_P - \hat{\beta}_{\text{OLS}}\bar{r}_m = 8.00\% - 1.4310(6.00\%) = -0.59\%$$

$$\beta_{\text{Blume}} = \frac{2}{3}(1.4310) + \frac{1}{3}(1.0000) = 0.9540 + 0.3333 = 1.2873$$

Using the Blume-adjusted beta of 1.29 at $R_f = 4.20\%$ and $\text{MRP} = 6.00\%$ yields an estimated cost of equity of $r_E = 4.20\% + 1.2873(6.00\%) = 11.92\%$ p.a.

3.5 Market Efficiency, Multifactor Extensions & Cost of Equity in Practice

Applying CAPM valuations and ASX market prices in corporate financial management relies on how rapidly and accurately capital markets incorporate new information. Fama (1970) classified the **Efficient Market Hypothesis (EMH)** into three nested informational tiers:

- **Weak-Form Efficiency:** Current share prices fully reflect all historical price and trading volume data; consequently, technical chart patterns and momentum rules cannot generate persistent risk-adjusted abnormal profits ($\alpha > 0$) after transaction costs.
- **Semi-Strong-Form Efficiency:** Current share prices rapidly and unbiasedly incorporate all publicly available information—including ASX earnings announcements, dividend declarations, macroeconomic releases from the RBA, and mergers—meaning fundamental analysis of public filings cannot consistently beat the market after adjusting for risk.
- **Strong-Form Efficiency:** Current share prices reflect all information, both public and private (insider information). Empirical evidence generally rejects strong-form efficiency, which explains why insider trading is strictly prohibited under Division 3 of Part 7.10 of the *Corporations Act 2001 (Cth)*.

Empirical event studies on the ASX confirm high semi-strong efficiency around continuous disclosure announcements (ASX Listing Rule 3.1): share prices adjust rapidly within minutes of earnings surprises, capital raisings, and M&A bids, leaving negligible risk-adjusted profits to delayed public traders. Nevertheless, behavioural finance documents systematic cognitive biases—including **overconfidence** (excessive trading and underestimated forecast intervals), the **disposition effect** (selling winning shares too early while holding losing shares too long to avoid realising paper losses), **anchoring** on historical 52-week highs, and institutional **herding**—that can create temporary mispricings when transaction costs, short-sale constraints, and fundamental risk limit arbitrage.

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EMH Tier	Information Set Reflected	Australian Capital Market Evidence
Weak Form	Historical share prices, returns, and trading volume.	Daily ASX return autocorrelations are near zero; simple moving-average rules fail after brokerage and market-impact costs.
Semi-Strong Form	All publicly available disclosures (ASX filings, RBA statements, financial reports).	ASX event studies show rapid price adjustment to dividend and earnings news, though minor post-earnings announcement drift persists in illiquid small caps.
Strong Form	All public and non-public (insider) information.	Rejected empirically; directors and insiders possess private information, justifying strict ASIC enforcement under *Corporations Act* Part 7.10.

Multifactor Models: Fama-French and Practical Cost of Equity

Empirical tests of the single-factor CAPM over several decades identified persistent return anomalies—notably that small-capitalisation stocks and low-market-to-book ("value") stocks earned higher average returns than predicted by market beta alone. Fama and French (1993) extended CAPM into a **Three-Factor Model** (later expanded by Carhart 1997 with momentum and Fama-French 2015 with profitability and investment):

$$E(R_i) - R_f = \beta_{i,M}\,[E(R_m) - R_f] + \beta_{i,\text{SMB}} E(\text{SMB}) + \beta_{i,\text{HML}} E(\text{HML})$$

Where: SMB (Small Minus Big) captures the size factor premium; HML (High Minus Low book-to-market) captures the value factor premium; and $\beta_{i,M}, \beta_{i,\text{SMB}}, \beta_{i,\text{HML}}$ are factor sensitivities.

Despite multifactor refinements, the single-factor CAPM remains the dominant tool among Australian corporate CFOs, investment banks, and economic regulators (such as the Australian Energy Regulator, AER) for estimating the cost of equity in capital budgeting. In Australian practice, practitioners match the risk-free rate maturity to the project horizon (typically the 10-year Commonwealth Government Bond yield) and pair it with a forward-looking or long-run historical Australian Market Risk Premium (typically 6.0%–6.5%, sometimes adjusted for dividend imputation credits as examined in Module 4).

Risk-Adjusted Portfolio Performance Evaluation

Asset pricing models also underpin institutional fund evaluation across Australia's superannuation sector. Because raw returns ignore differences in risk taken, performance attribution relies on three classic risk-adjusted ratios:

- **Sharpe Ratio:** Excess return per unit of *total* volatility ($\frac{\bar{R}_p - \bar{R}_f}{\sigma_p}$), appropriate when evaluating an investor's entire risky portfolio or comparing funds that serve as the sole risky holding.
- **Treynor Ratio:** Excess return per unit of *systematic* risk ($\frac{\bar{R}_p - \bar{R}_f}{\beta_p}$), appropriate when evaluating an individual active sleeve or sub-portfolio held inside a broadly diversified multi-manager superannuation fund.
- **Jensen's Alpha (α_p):** Realised excess return above the CAPM SML benchmark: $\alpha_p = \bar{R}_p - [\bar{R}_f + \beta_p(\bar{R}_m - \bar{R}_f)]$.

Worked Example 6: Fama-French Three-Factor Cost of Equity and Fund Attribution

Part A — Fama-French Cost of Equity: Suppose an ASX small-cap value retailer has estimated factor loadings of $\beta_{i,M} = 1.10$ (market), $\beta_{i,\text{SMB}} = 0.65$ (size), and $\beta_{i,\text{HML}} = 0.45$ (value). Given $R_f = 4.20\%$, $\text{MRP} = 6.00\%$, expected size premium $E(\text{SMB}) = 2.40\%$, and expected value premium $E(\text{HML}) = 3.20\%$:

$$E(R_i) = 4.20\% + 1.10(6.00\%) + 0.65(2.40\%) + 0.45(3.20\%)$$

$$E(R_i) = 4.20\% + 6.60\% + 1.56\% + 1.44\% = 13.80\%\ \text{p.a.}$$

Whereas the single-factor CAPM would assign a cost of equity of $r_{E,\text{CAPM}} = 4.20\% + 1.10(6.00\%) = 10.80\%$, the Fama-French model adds 3.00 percentage points for the firm's exposure to small-capitalisation and high book-to-market risk factors.

Part B — Superannuation Fund Performance Attribution: Over a 5-year evaluation period, the S&P/ASX 200 returned $\bar{R}_m = 10.50\%$ ($\sigma_m = 15.0\%$) and the average risk-free rate was $\bar{R}_f = 3.50\%$. Two Australian active equity funds recorded the following statistics:

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Metric	Fund A (Concentrated Growth)	Fund B (Diversified Core)	ASX 200 Benchmark
Average Return ($\bar{R}_p$)	12.80%	11.40%	10.50%
Total Volatility (σ_p)	21.00%	14.50%	15.00%
Portfolio Beta (β_p)	1.25	0.95	1.00
Sharpe Ratio	0.443	0.545	0.467
Treynor Ratio	7.44%	8.32%	7.00%
Jensen's Alpha (α_p)	+0.55%	+1.25%	0.00%

Even though Fund A achieved a higher raw return (12.80% vs 11.40%), Fund B outperformed on every risk-adjusted metric: its Sharpe Ratio (0.545) beat the market (0.467), whereas Fund A's Sharpe Ratio (0.443) lagged the benchmark due to uncompensated idiosyncratic concentration risk, and Fund B delivered higher Jensen's alpha (+1.25% vs +0.55%).

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