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Bachelor of Commerce (B2001) | Level 2 • Part B Major | Monash University

Introduction to Management Accounting

Course Code: ACC2200 • Module 3 Notes

Chapter 1: Activity-Based Costing (ABC) and Overhead Distortion

1.1 Cross-Subsidisation and the Death Spiral in Traditional Costing

Traditional volume-based costing systems—whether using a single plant-wide rate or broad departmental rates—allocate indirect manufacturing overhead to products exclusively through unit-level volume drivers such as direct labour hours, machine hours, or direct material dollars. When modern Australian manufacturers and service providers offer diverse product lines ranging from high-volume standardised items to low-volume customised batches, unit-level drivers systematically distort product costs.

This distortion arises because many overhead resources—such as CNC machine setups, engineering change orders, supplier quality audits, and dispatch scheduling—are consumed in proportion to **transactions, batches, and product complexity** rather than physical unit volume. A low-volume specialty order of 20 units may trigger the same machine setup and quality inspection effort as a high-volume run of 2,000 standard units. Allocating setup and inspection overhead on direct labour hours shifts costs away from complex low-volume items onto simple high-volume items, a phenomenon known as **product cross-subsidisation**:

- **High-volume simple products are over-costed:** Competitors with focused factories undercut the firm's inflated prices, eroding market share on core cash-cow lines.
- **Low-volume complex products are under-costed:** Unaware of true batch and sustaining costs, sales teams win unprofitable custom tenders, creating a **pricing death spiral** as fixed overheads are spread across shrinking core volumes.

1.2 Two-Stage ABC Architecture and the Cost Hierarchy

Activity-Based Costing (ABC) eliminates volume distortion by modelling causal resource consumption across two allocation stages:

1. **Stage 1 (Resources to Activity Cost Pools):** General ledger expenses (wages, depreciation, software licences, utilities) are assigned to distinct **activity cost pools** (e.g., Machine Setup, Order Processing, Quality Testing) using **resource drivers** (e.g., staff time surveys, floor area, kilowatt metering).
2. **Stage 2 (Activity Cost Pools to Cost Objects):** Each activity pool is divided by its practical capacity or budgeted volume of its **activity cost driver** to obtain an activity driver rate (ADR_k), which is then traced to products, jobs, or customers based on actual driver usage:

$$ADR_k = \frac{\text{Total Budgeted Cost of Activity Pool } k}{\text{Total Practical Activity Driver Volume } k} \qquad \text{and} \qquad OH_j = \sum_{k=1}^{m} (ADR_k \times d_{jk})$$

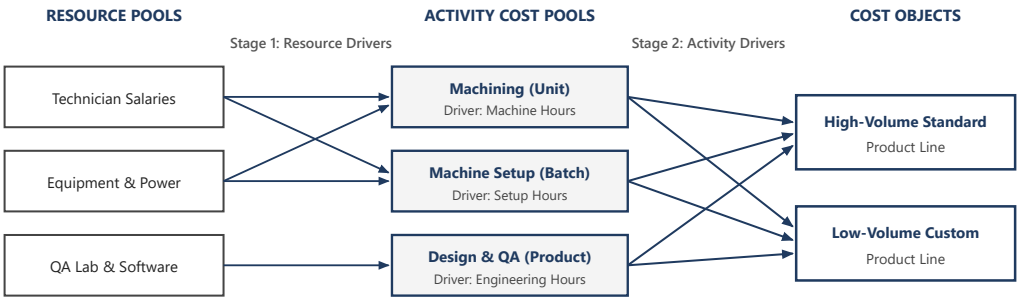
Where:

- ADR_k: Activity Driver Rate for activity cost pool k (AUD per driver transaction or hour).
- OH_j: Total indirect overhead allocated to cost object j across all m activity pools.
- d_{jk}: Quantity of activity driver k consumed by cost object j.

ABC classifies activities along a four-tier **cost hierarchy** so managers never erroneously treat batch or product-level costs as variable with individual units:

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Hierarchy Level	Causal Relationship	Representative Activity Pools & Drivers
1. Unit-Level Activities	Performed every time an individual unit is produced or service delivered.	CNC machining power (machine hours), automated welding (weld minutes), unit packaging (units).
2. Batch-Level Activities	Performed once per production batch, purchase order, or shipment, regardless of batch size.	Machine setup & calibration (setup hours), material handling (moves), supplier purchase orders (POs).
3. Product-Sustaining Activities	Performed to support the existence and compliance of a specific product SKU or service line.	Engineering design updates (engineering hours), TGA/AS quality certification (testing hours), BOM maintenance.
4. Facility-Sustaining Activities	Support overall organisational infrastructure; cannot be traced causally to individual products.	Plant security, factory council rates, general executive management (typically excluded from ABC unit rates or allocated only for full absorption).



1.3 Worked Example 3.1: Traditional Costing Versus ABC

Adelaide Medical Devices Pty Ltd manufactures two surgical instruments: **Standard Forceps (SF-10)** and **Custom Titanium Retractors (TR-90)**. Annual budgeted manufacturing overhead totals $960,000, currently allocated on 32,000 total machine hours ($POHR$ = $960,000 / 32,000 = $30.00 per MH). An ABC team partitions the $960,000 overhead into three activity pools:

Activity Pool	Pool Cost (AUD)	Cost Driver	SF-10 Usage	TR-90 Usage	Total Driver Vol.	Activity Rate (ADR)
Machining (Unit)	$384,000	Machine Hours (MH)	24,000 MH	8,000 MH	32,000 MH	$12.00 / MH
Machine Setup (Batch)	$336,000	Production Batches	40 batches	200 batches	240 batches	$1,400.00 / batch
Quality & Engineering (Product)	$240,000	Engineering Hours	400 EH	1,600 EH	2,000 EH	$120.00 / EH
Total Overhead	**$960,000**	—	—	—	—	—

Production volume is 10,000 units of SF-10 (prime cost $42.00 per unit) and 2,000 units of TR-90 (prime cost $95.00 per unit).

Step 1: Traditional Volume-Based Overhead per Unit

- **SF-10:** (24,000 MH × $30.00) / 10,000 units = $720,000 / 10,000 = $72.00 OH per unit. Total unit cost = $42.00 + $72.00 = $114.00.

- **TR-90:** (8,000 MH × $30.00) / 2,000 units = $240,000 / 2,000 = $120.00 OH per unit. Total unit cost = $95.00 + $120.00 = $215.00.

Step 2: ABC Overhead and Total Unit Cost

- **SF-10 Total ABC Overhead:** (24,000 × $12.00) + (40 × $1,400) + (400 × $120) = $288,000 + $56,000 + $48,000 = $392,000 → $39.20 per unit. Total ABC unit cost = $42.00 + $39.20 = **$81.20**.
- **TR-90 Total ABC Overhead:** (8,000 × $12.00) + (200 × $1,400) + (1,600 × $120) = $96,000 + $280,000 + $192,000 = $568,000 → $284.00 per unit. Total ABC unit cost = $95.00 + $284.00 = **$379.00**.

Traditional costing over-costed high-volume SF-10 by $32.80 per unit ($114.00 − $81.20) and severely under-costed complex low-volume TR-90 by $164.00 per unit ($379.00 − $215.00). If TR-90 was priced at $280 based on the $215 traditional cost, Adelaide Medical Devices actually lost $99.00 on every retractor sold.

1.4 Time-Driven ABC (TDABC) and Customer Profitability Analysis

Conventional ABC often requires expensive employee time-allocation surveys that become outdated as processes evolve and fail to distinguish used from unused capacity. **Time-Driven Activity-Based Costing (TDABC)** eliminates subjective surveys using only two parameters: the **Capacity Cost Rate (CCR)** of a department and linear **time equations** estimating the minutes required for each transaction variant:

$$CCR = \frac{\text{Total Cost of Capacity Supplied}}{\text{Practical Capacity of Staff or Equipment (in Minutes)}}$$

Where:

- CCR: Cost per minute of productive departmental capacity (AUD/min).
- $\text{Total Cost of Capacity Supplied}$: Departmental wages, supervision, equipment depreciation, software, and occupancy costs.
- $\text{Practical Capacity}$: Typically 80%–85% of theoretical paid hours, excluding breaks, training, and routine maintenance downtime.

Worked Example 3.2: TDABC Capacity Rate and Unused Capacity

Melbourne Commercial Logistics Pty Ltd operates a customer-order fulfilment desk costing $54,000 per month with 6 full-time clerks (each paid 160 hours/month, with 80% practical availability = 128 productive hours, or 7,680 minutes per clerk). Total practical capacity = 6 × 7,680 = 46,080 minutes/month. Thus:

- $CCR = \frac{54{,}000}{46{,}080} = 1.1719 \text{ AUD/min}$ ($1.1719 per minute).
- Suppose the order-processing time equation is $T = 8 + 12(\text{if new customer}) + 5(\text{if cold-chain}) + 2(\text{line items})$. During September, the desk processed 1,800 standard orders (averaging 14 min each = 25,200 min) and 450 cold-chain rush orders (averaging 26 min each = 11,700 min), consuming 36,900 total minutes.
- **Cost Assigned to Orders:** 36,900 min × $1.171875/min = $43,242.19.
- **Cost of Unused Capacity Exposed:** $(46{,}080 - 36{,}900) = 9{,}180$ unused minutes × $1.171875 = $10,757.81. Rather than burying this $10,758 idle cost inside higher order charges, TDABC alerts managers that capacity exists to absorb 20% volume growth or redeploy one clerk.

Extending ABC/TDABC to selling, logistics, and customer-service expenses supports **Customer Profitability Analysis (CPA)**. Plotting cumulative operating profit across customers ranked from highest to lowest net profitability commonly produces a **whale curve**: the top 20% of customers often generate 150%–250% of total

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corporate profit, middle-tier accounts break even, and the bottom 20% destroy substantial value due to excessive small orders, custom packaging, rush couriers, and warranty claims.

Worked Example 3.2B: Customer Profitability and Cost-to-Serve Analysis

Consider three wholesale accounts served by *Melbourne Commercial Logistics Pty Ltd*, each generating $120,000 in annual gross revenue and $42,000 in gross product margin (35%). Using TDABC cost-to-serve rates ($45 per standard order, $180 per rush air-freight dispatch, $120 per sales-rep visit, and $95 per credit-note return):

Profitability Metric	Customer Alpha (Predictable)	Customer Beta (Moderate)	Customer Gamma (High-Maintenance)
Annual Sales Revenue	$120,000	$120,000	$120,000
Product Gross Margin (35%)	$42,000	$42,000	$42,000
Standard Orders (@ $45)	24 ($1,080)	80 ($3,600)	220 ($9,900)
Rush Dispatches (@ $180)	2 ($360)	15 ($2,700)	95 ($17,100)
Field Rep Visits (@ $120)	4 ($480)	12 ($1,440)	60 ($7,200)
Returns & Credit Notes (@ $95)	1 ($95)	8 ($760)	110 ($10,450)
Total Cost-to-Serve	**$2,015**	**$8,500**	**$44,650**
Net Customer Profit (Loss)	**$39,985 (33.3%)**	**$33,500 (27.9%)**	**($2,650) (-2.2%)**

Although all three clients deliver identical gross margins ($42,000), Customer Gamma destroys value through fragmented small orders, rush freight, and chronic returns. Rather than terminating Gamma immediately, management can introduce minimum order thresholds, unbundle express courier fees, and offer electronic EDI ordering discounts—converting an unprofitable account into a profitable partner.

Chapter 2: Cost Behaviour Estimation and Quantitative Cost Functions

2.1 Linear Cost Functions and Estimation Criteria

To construct budgets, flexible overhead formulas, and CVP projections, managers must decompose mixed costs into fixed and variable parameters using a linear approximation within the relevant range:

$$Y = a + bX$$

Where:

- Y: Predicted total mixed cost (dependent variable, in AUD).
- a: Fixed cost intercept (cost incurred at zero activity within the relevant range).
- b: Variable cost slope (incremental cost per unit change in activity driver X).
- X: Activity level of the independent cost driver (e.g., machine hours, kilometres driven, orders shipped).

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Two criteria govern cost-function selection: **economic plausibility** (a sound operational cause-and-effect link between X and Y) and **goodness-of-fit** (how closely the estimated line tracks historical cost observations). Prior to quantitative fitting, analysts must also adjust historical cost records for general inflation, price-level discontinuities, and accounting accrual lags (so maintenance expenses incurred in June are matched with June machine hours rather than July invoice dates).

2.2 Qualitative and Quantitative Estimation Methods

Four primary methods are used to estimate $Y = a + bX$:

1. **Account Analysis:** Classifying each general ledger sub-account (e.g., power, lubricants, supervisor pay) as fixed, variable, or mixed based on managerial judgement and experience. Fast and intuitive, but subjective.
2. **Engineering (Work-Measurement) Method:** Analysing physical input-output relationships through time-and-motion studies and material specifications. Ideal for new products lacking historical data, though costly to conduct across indirect tasks.
3. **High-Low Method:** Fits a straight line exclusively through the observations with the *highest* and *lowest* values of the cost driver X (not Y):

$$b_{\text{HL}} = \frac{Y_{\text{high}} - Y_{\text{low}}}{X_{\text{high}} - X_{\text{low}}} \qquad \text{and} \qquad a_{\text{HL}} = Y_{\text{high}} - b_{\text{HL}} X_{\text{high}}$$

Where:

- b_{HL}: Estimated variable cost per unit of activity driver.
- a_{HL}: Estimated fixed cost intercept.
- $(X_{\text{high}}, Y_{\text{high}})$ and $(X_{\text{low}}, Y_{\text{low}})$: Data pairs selected strictly by maximum and minimum activity X.

While easy to compute, the High-Low method discards all intermediate observations and is acutely vulnerable to outlier distortion if either endpoint reflects an abnormal equipment breakdown or accounting accrual error.

4. **Ordinary Least Squares (OLS) Linear Regression:** Uses every valid observation (X_i, Y_i) across n periods to minimise the sum of squared vertical residuals $\sum (Y_i - \hat{Y}_i)^2$:

$$b = \frac{n\sum X_i Y_i - (\sum X_i)(\sum Y_i)}{n\sum X_i^2 - (\sum X_i)^2} \qquad \text{and} \qquad a = \bar{Y} - b\bar{X}$$

Where:

- b: OLS slope coefficient (variable cost per driver unit); a: OLS intercept (fixed cost).
- n: Number of historical observations; $\bar{X}$ and $\bar{Y}$: Sample means of activity and cost.

Managers evaluate OLS regressions using four statistical diagnostics:

- **Coefficient of Determination (R^2):** Proportion of total variance in cost Y explained by variation in driver X (ranging from 0 to 1, i.e., $R^2 \in [0, 1]$). Values above 0.70 generally indicate strong linear explanatory power.
- **Standard Error of the Estimate (s_e):** Typical dispersion of actual costs around the regression line; smaller values indicate tighter prediction intervals.
- ***t*-Statistic of the Slope ($t_b = b/SE_b$):** Tests whether b differs significantly from zero; a $|t_b| > 2.0$ (at 95% confidence for moderate sample sizes) confirms a statistically reliable driver relationship.

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- **Multiple Regression & Specification Checks:** When two or more cost drivers operate simultaneously (e.g., machine hours X_1 and production batches X_2), multiple regression estimates $\hat{Y} = a + b_1X_1 + b_2X_2$. Analysts must verify that independent variables do not suffer from severe **multicollinearity** (high inter-correlation between X_1 and X_2 inflating standard errors) and that residuals display constant variance (homoscedasticity) without serial autocorrelation.

2.3 Worked Example 3.3: High-Low Versus OLS Regression

Hunter Valley Logistics Pty Ltd tracks monthly heavy-fleet maintenance cost (Y) and fleet kilometres driven (X, in thousands of km) over 6 recent months (adjusted for inflation):

Month	Km Driven (X, '000 km)	Maintenance Cost (Y, AUD)	X^2	XY
January	40	$38,000	1,600	1,520,000
February	50	$43,500	2,500	2,175,000
March	65	$52,000	4,225	3,380,000
April	80	$59,000	6,400	4,720,000
May	55	$45,500	3,025	2,502,500
June (High X)	90	$68,000	8,100	6,120,000
Sum ($\sum$, $n = 6$)	**380**	**$306,000**	**25,850**	**20,417,500**

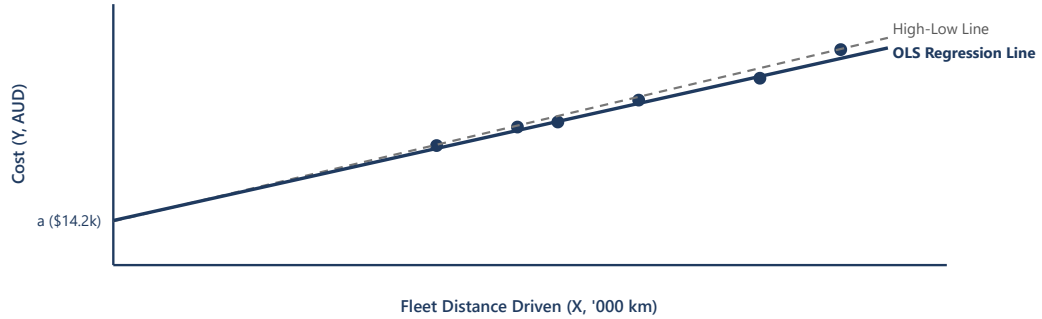
Step 1: High-Low Method

- Highest activity: June ($X_{\text{high}} = 90$, Y_{high} = $68,000). Lowest activity: January ($X_{\text{low}} = 40$, Y_{low} = $38,000).
- $b_{\text{HL}} = \frac{68,000 - 38,000}{90 - 40} = \frac{30,000}{50}$ = $600.00 per '000 km (or $0.60 per km).
- a_{HL} = $68,000 – ($600 × 90) = $14,000 per month.
- High-Low Cost Function: $\hat{Y}$ = $14,000 + 600X$.

Step 2: OLS Linear Regression

- $\bar{X} = 380/6 = 63.3333$; $\bar{Y}$ = $306,000 / 6 = $51,000.
- Numerator of b: $6(20,417,500) - (380)(306,000) = 122,505,000 - 116,280,000 = 6,225,000$.
- Denominator of b: $6(25,850) – (380)^2$ = 155,100 – 144,400 = 10,700.
- $b = \frac{6,225,000}{10,700}$ = $581.78 per '000 km ($0.5818 per km).
- a = $51,000 – ($581.7757 × 63.3333) = $51,000 – $36,845.79 = $14,154.21.
- OLS Cost Function: $\hat{Y}$ = $14,154 + 581.78X$ (with $R^2 = 0.990$, confirming 99.0% of monthly cost variance is driven by kilometres).

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2.4 Non-Linear Labour Estimation: Learning Curves

In custom engineering, defence shipbuilding, and complex assembly, direct labour hours per unit decline systematically as workers gain experience. Under the **cumulative average-time learning model**, every time cumulative output doubles, cumulative average time per unit drops to a constant learning percentage r (e.g., 80%):

$$y = ax^b \qquad \text{where} \qquad b = \frac{\ln(r)}{\ln(2)}$$

Where:

- y: Cumulative average labour hours per unit across x cumulative units.
- a: Direct labour hours required to produce the very first unit ($x = 1$).
- x: Cumulative number of units produced.
- b: Learning index (negative constant derived from learning rate r, e.g., for $r = 0.80$, $b = \ln(0.80)/\ln(2) \approx -0.3219$).

Worked Example 3.4: 80% Cumulative Average-Time Learning Curve

Tasman Defence Fabrication Pty Ltd wins a tender to assemble 8 specialised sonar housings. The first unit requires $a = 200$ direct labour hours, and operations follow an $r = 80\%$ learning curve ($b = -0.321928$), with loaded labour costing $65.00 per hour.

Cumulative Units (x)	Cumulative Avg Hours/Unit (y)	Cumulative Total Hours ($x \cdot y$)	Incremental Hours for Batch	Cumulative Labour Cost (@ $65/hr)
1	200.00 hrs	200.00 hrs	200.00 hrs (Unit 1)	$13,000
2	200 × 0.80 = 160.00 hrs	320.00 hrs	120.00 hrs (Unit 2)	$20,800
4	160 × 0.80 = 128.00 hrs	512.00 hrs	192.00 hrs (Units 3–4)	$33,280
8	128 × 0.80 = 102.40 hrs	819.20 hrs	307.20 hrs (Units 5–8)	$53,248

Notice that ignoring learning effects and pricing all 8 units at the initial 200-hour rate (8 × 200 × $65 = $104,000) would overstate labour cost by $50,752 and cause the firm to lose the tender.

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Chapter 3: Relevant Information and Short-Term Tactical Decision-Making

3.1 Defining Relevant Costs and Revenues

Short-term tactical decisions operate within existing fixed capacity over horizons typically under one year. Rather than relying on full absorption unit costs—which blend variable costs, sunk historical outlays, and arbitrary corporate overhead allocations—managers apply **differential (incremental) analysis**, focusing strictly on **relevant information**. A cost or revenue is relevant if and only if it satisfies two conditions simultaneously:

1. **Future-Oriented:** It occurs in the future; past outlays already incurred are **sunk costs** and are strictly irrelevant because no current choice can alter them (e.g., carrying book value or historical depreciation of existing equipment).
2. **Differs Across Alternatives:** Any future cost or revenue that remains identical under every considered option (such as unavoidable corporate head-office rent) cancels out in the comparison and is irrelevant.

Conversely, an **opportunity cost**—the contribution margin forgone by rejecting the next-best alternative use of a scarce resource (such as factory floor space or machine hours)—is always relevant even though it never appears as a cash disbursement in the general ledger.

3.2 Four Tactical Decision Archetypes (With Worked Examples)

1. Special Order Decisions

A special order involves a one-off request from a non-regular customer at a discounted selling price. When the plant holds **idle capacity**, any special-order price exceeding strictly incremental variable costs plus incremental batch/setup costs increases operating profit. At **full capacity**, the firm must also recover the opportunity cost (forgone contribution margin on displaced regular sales). Beyond quantitative margins, managers must check Australian competition law guardrails under the *Competition and Consumer Act 2010 (Cth)* (avoiding predatory pricing) and ensure regular customers will not demand matching discounts.

Worked Example 3.5 (Special Order): *Tasman Bottling Pty Ltd* manufactures commercial glass bottles selling normally at $2.80 per unit. Unit costs at normal volume (100,000 units) are: DM $0.85, DL $0.55, Variable MOH $0.30, Fixed MOH $0.60 (total $2.30). An export distributor offers a one-off order for 15,000 bottles at $1.95 each, requiring a custom export label costing $1,800 total. Tasman currently operates at 80,000 units (capacity 100,000 units, so 20,000 idle capacity exists; no regular sales displaced). Incremental revenue = 15,000 × $1.95 = $29,250. Incremental costs = 15,000 × ($0.85 + $0.55 + $0.30) + $1,800 = $25,500 + $1,800 = $27,300. Fixed MOH ($0.60) is irrelevant because total fixed overhead does not change. Net incremental profit = $29,250 − $27,300 = **+$1,950**; accept the order.

2. Make-or-Buy (Insourcing vs Outsourcing) Decisions

When deciding whether to manufacture a sub-component internally or purchase it from an external supplier, only **avoidable** costs of internal production are relevant. Unavoidable allocated factory overhead continues even if outsourced.

Worked Example 3.6 (Make-or-Buy): *Fremantle Marine Pumps Pty Ltd* produces 10,000 brass housings annually at an absorption cost of $46.00 each: DM $16.00, DL $12.00, Variable MOH $5.00, Supervisor salary (avoidable if outsourced) $4.00, and Allocated Plant Depreciation (unavoidable) $9.00. An external supplier quotes $39.00 per housing. If outsourced, the freed assembly bay can be leased to an adjacent business for $35,000 per year.

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- Relevant cost to Make: 10,000 × ($16 + $12 + $5 + $4) = $370,000 (or $37.00/unit).
- Relevant cost to Buy (net of opportunity benefit): (10,000 × $39.00) − $35,000 lease income = $390,000 − $35,000 = $355,000.
- Net advantage to Buy = $370,000 − $355,000 = **+$15,000** per year. Before outsourcing, management must also verify supplier delivery reliability, IP protection, and supply-chain ethical compliance (including obligations under the Australian *Modern Slavery Act 2018 (Cth)* for large entities).

3. Keep-or-Drop a Product Line or Segment

Income statements that allocate common corporate costs across divisions often make marginal segments appear unprofitable. Dropping a segment eliminates its revenue, variable costs, and **avoidable direct fixed costs**, but common unavoidable fixed costs remain and are merely reallocated onto surviving segments.

Worked Example 3.7 (Keep-or-Drop): A Sydney retail chain evaluates closing its Parramatta store, which reports Sales $420,000, Variable COGS & commissions $260,000 (CM = $160,000), Store Manager salary & store lease (avoidable) $135,000, and Allocated Head-Office IT/Executive Costs (unavoidable) $45,000—showing an accounting loss of ($20,000). However, Parramatta's **segment margin** is $160,000 − $135,000 = **+$25,000**. Closing the store would sacrifice $25,000 of positive contribution toward head-office overheads, reducing total company profit by $25,000 unless the closure boosts online cannibalised recapture or frees capital for higher-return sites.

4. Sell-at-Split-Off Versus Process-Further (Joint Products)

In timber milling, meat processing, and petroleum refining, two or more **joint products** emerge simultaneously from a common input at the **split-off point**. All joint processing costs incurred prior to split-off are **sunk costs** with respect to post-split-off decisions. A product should be processed further whenever incremental revenue after split-off exceeds separable (incremental) further-processing costs:

> $$\text{Incremental Profit}_{\text{further}} = (\text{Final Sales Value} - \text{Split-Off Sales Value}) - \text{Separable Processing Cost}$$

Where:

- Final Sales Value: Total revenue obtainable after completing secondary processing.
- Split-Off Sales Value: Immediate market revenue obtainable at the split-off point without further work.
- Separable Processing Cost: Incremental materials, labour, and avoidable overhead incurred strictly beyond split-off.

Worked Example 3.8 (Joint Processing): *Wagga Abattoirs Ltd* incurs $120,000 in joint processing costs to obtain 5,000 kg of Prime Cut (worth $65,000 at split-off) and 3,000 kg of Trim (worth $18,000 at split-off). Trim can be processed further into gourmet sausages at a separable cost of $14,000 and sold for $38,000. Incremental revenue = $38,000 − $18,000 = $20,000. Subtracting separable processing cost of $14,000 yields an incremental profit of **+$6,000**. The $120,000 joint cost is sunk and completely irrelevant.

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Chapter 4: Constrained Resources, Bottleneck Management and Linear Programming Intuition

4.1 Single-Constraint Optimisation

When a firm faces a single binding bottleneck—such as skilled welder hours, autoclave curing time, or CNC spindle hours—maximising total operating profit does *not* mean prioritising the product with the highest contribution margin per unit (CM_u). Instead, management must maximise the **Contribution Margin per Unit of the Constrained Resource**:

$$CM_{\text{constraint},i} = \frac{CM_{u,i}}{r_i}$$

Where:

- $CM_{\text{constraint},i}$: Contribution margin generated per unit of the bottleneck resource by product i (AUD per constraint hour).
- $CM_{u,i}$: Unit contribution margin of product i ($P_i - V_i$).
- r_i: Quantity of the constrained resource required to produce one unit of product i.

Worked Example 3.9: Single-Constraint Production Schedule

Perth Precision Fabrication Pty Ltd has a maximum of **2,400 CNC machine hours (MH)** available per month. Three components are in demand:

Metric	Model X (Standard)	Model Y (Heavy)	Model Z (Ultra)
Unit Contribution Margin (CM_u)	\$90	\$140	\$160
CNC Hours Required per Unit (r_i)	1.5 MH	2.8 MH	4.0 MH
CM **per CNC Hour (CM_u/r_i)**	**\$60.00 / MH (Rank 1)**	**\$50.00 / MH (Rank 2)**	**\$40.00 / MH (Rank 3)**
Maximum Monthly Market Demand	800 units	500 units	300 units

Although Model Z earns the highest CM_u (\$160), it delivers the lowest return per bottleneck hour (\$40/MH). Optimal allocation of the 2,400 MH proceeds strictly by rank:

1. Produce full demand of Rank 1 (Model X): 800 units × 1.5 MH = 1,200 MH (remaining = 1,200 MH).
2. Produce as much Rank 2 (Model Y) as possible: 1,200 MH / 2.8 MH per unit = 428.57 → 428 whole units (1,198.4 MH used).
3. Produce 0 units of Model Z. Maximum total contribution margin = (800 × \$90) + (428 × \$140) = \$72,000 + \$59,920 = **\$131,920**.

Furthermore, management knows it should pay up to **\$50.00 per hour** (the marginal value of the bottleneck—its shadow price) to outsource Model Y machining or run overtime on the CNC machine to satisfy the remaining 72 units of Model Y demand.

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4.2 Multiple Constraints and Linear Programming (LP) Intuition

When two or more resource constraints bind simultaneously (e.g., both CNC Machining hours and Hand Finishing hours), simple ratio ranking fails because different products stress different bottlenecks. Managers formulate a **Linear Programming (LP)** model comprising an objective function, structural resource constraints, and non-negativity conditions:

$$\max Z = \sum_{i=1}^{n} CM_{u,i}x_i \qquad \text{subject to} \qquad \sum_{i=1}^{n} a_{ki}x_i \leq b_k \quad (\forall k), \quad x_i \geq 0$$

Where:

- Z: Total portfolio contribution margin (AUD) to be maximised.
- x_i: Optimal production quantity of product i.
- a_{ki}: Units of constrained resource k consumed per unit of product i.
- b_k: Total available capacity of resource k.

By the **Corner-Point Theorem**, the optimal LP solution always lies at an extreme vertex (corner point) of the convex **feasible region** formed by intersecting constraints. A constraint with zero unused capacity at the optimum is **binding** and possesses a positive **shadow price** (the increase in maximum Z per additional unit of b_k); a constraint with unused capacity (**slack**) has a shadow price of zero.

Worked Example 3.10: Two-Product Linear Programming Solution

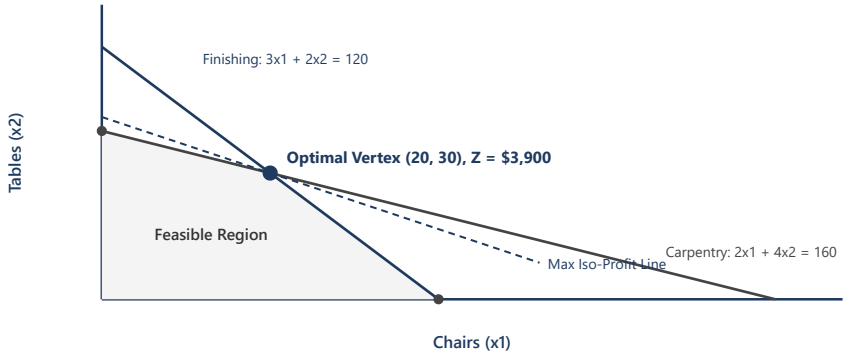
Suppose a workshop makes Chairs (x_1, CM_1 = \$60) and Tables (x_2, CM_2 = \$90), subject to Carpentry and Finishing capacity limits:

$$\text{Carpentry: } 2x_1 + 4x_2 \leq 160 \text{ hrs}, \qquad \text{Finishing: } 3x_1 + 2x_2 \leq 120 \text{ hrs}, \qquad x_1, x_2 \geq 0$$

- **Corner 1 (Origin):** $(0,0) \implies Z$ = \$0.
- **Corner 2 (Finishing x_1-intercept):** $(40,0) \implies Z = 60(40)$ = \$2,400.
- **Corner 3 (Carpentry x_2-intercept):** $(0,40) \implies Z = 90(40)$ = \$3,600.
- **Corner 4 (Intersection of both binding constraints):** Multiply Finishing by 2 to obtain $6x_1 + 4x_2 = 240$ and subtract Carpentry ($2x_1 + 4x_2 = 160$) to get $4x_1 = 80$, giving $x_1 = 20$. Substituting back gives $x_2 = 30$. Thus $(20,30) \implies Z = 60(20) + 90(30)$ = \$1,200 + \$2,700 = **\$3,900** (Optimal!).

Sensitivity and Shadow Price Check: If management secures 16 additional Carpentry hours (raising b_1 from 160 to 176), solving the updated system ($2x_1 + 4x_2 = 176$ and $3x_1 + 2x_2 = 120$) shifts the optimal vertex to $(x_1 = 16, x_2 = 36)$, raising Z to 60(16) + 90(36) = \$4,200—an increase of \$300 across 16 hours, meaning Carpentry's shadow price is \$300 / 16 = **\$18.75 per hour** within the allowable range of the basis.

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4.3 Theory of Constraints (TOC) and Throughput Accounting

Complementing linear programming, Goldratt's **Theory of Constraints (TOC)** views short-term labour and facility expenses as committed fixed costs and seeks to maximise system cash generation via three operational metrics: **Throughput (T)** (Sales Revenue minus strictly Direct Materials and outsourced piecework), **Inventory/Investment (I)** (money tied up in raw materials, WIP, and equipment), and **Operating Expense (OE)** (all conversion and overhead outlays required to turn Inventory into Throughput). TOC manages bottlenecks through five continuous focusing steps:

1. **Identify the System Constraint:** Locate the physical workstation, skilled labour pool, or market demand limit dictating overall plant throughput.
2. **Exploit the Constraint:** Eliminate downtime, enforce pre-bottleneck quality inspections (so defective parts never waste bottleneck minutes), and prioritise products with the highest *Throughput per Bottleneck Minute* ($T/\text{Bottleneck Min}$).
3. **Subordinate Non-Bottlenecks:** Synchronise upstream departments to the bottleneck's exact pace (the Drum-Buffer-Rope system), preventing excess WIP buildup.
4. **Elevate the Constraint:** Expand bottleneck capacity via capital investment, cross-training, or targeted subcontractor outsourcing.
5. **Prevent Inertia:** Once a constraint breaks, return immediately to Step 1 because a new workstation elsewhere in the factory has become the binding limit.

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